

# Multiscale Study of Hypersonic Vehicles: From Turbulence to Ceramics

James Chen, Ph.D.

Associate Professor  
Multiscale Computational Physics Laboratory (MCPL)  
Department of Mechanical and Aerospace Engineering  
Center for Hybrid Rocket Exascale Simulation Technology (CHREST)  
University at Buffalo - The State University of New York

Webinar with DSIAC  
December 13, 2023

# About Me



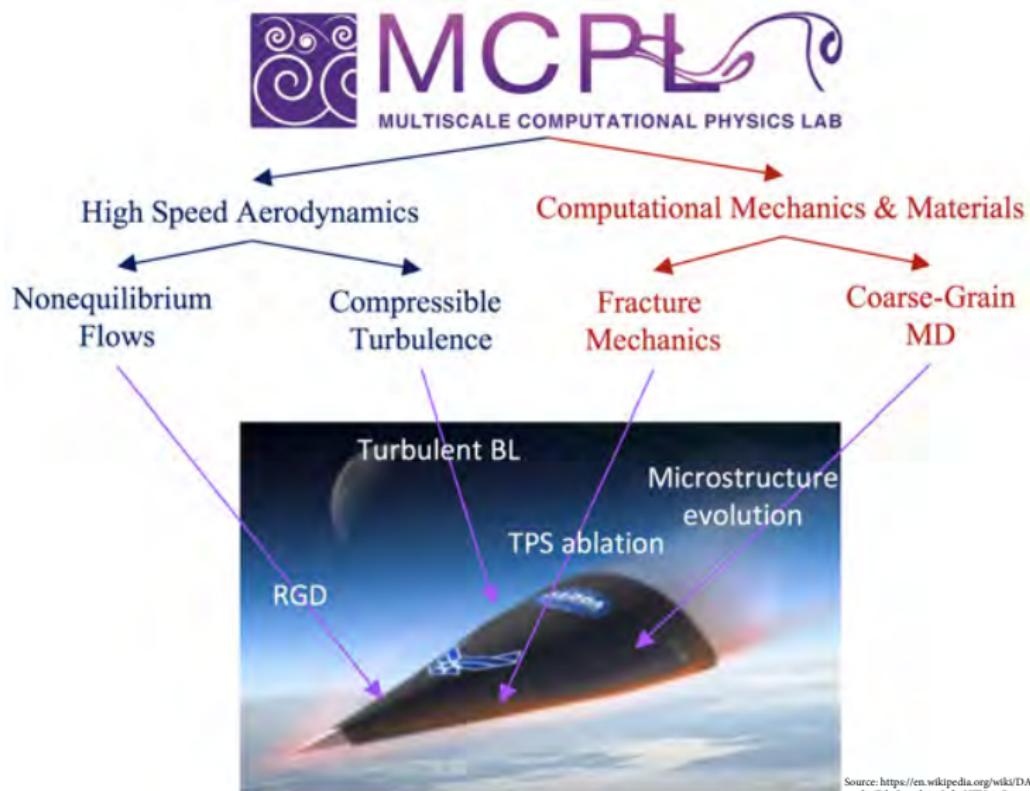
## Current Position

- Associate Professor of Mechanical and Aerospace Engineering
- Undergraduate Director of Aerospace Engineering

## Selected Honors & Awards

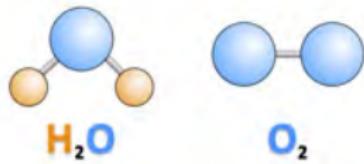
- Fellow of ASME, 2022
- Associate Fellow of AIAA, 2022
- Outstanding Young Engineer Award, AIAA Wichita Section, 2018
- Young Investigator Award, AFOSR, 2017

# Research at MCPL



Source: [https://en.wikipedia.org/wiki/DARPA\\_Falcon\\_Project#/media/File:Speed\\_is\\_Life\\_HTV-2\\_Reentry\\_New.jpg](https://en.wikipedia.org/wiki/DARPA_Falcon_Project#/media/File:Speed_is_Life_HTV-2_Reentry_New.jpg)

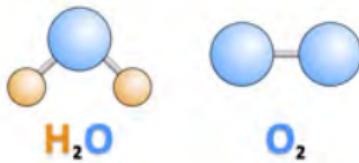
# Rotation or Not?



Source: <https://www.flickr.com/photos/cseeman/27442272417>

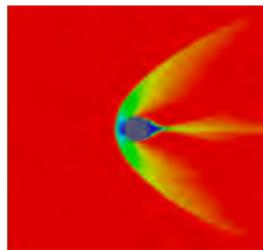
Boltzmann Equation and Navier-Stokes Equations are for **Monatomic Gas** or **Volumeless Point Mass**.

# Rotation or Not?



Source: <https://www.flickr.com/photos/cseeman/27442272417>

Boltzmann Equation and Navier-Stokes Equations are for **Monatomic Gas** or **Volumeless Point Mass**.

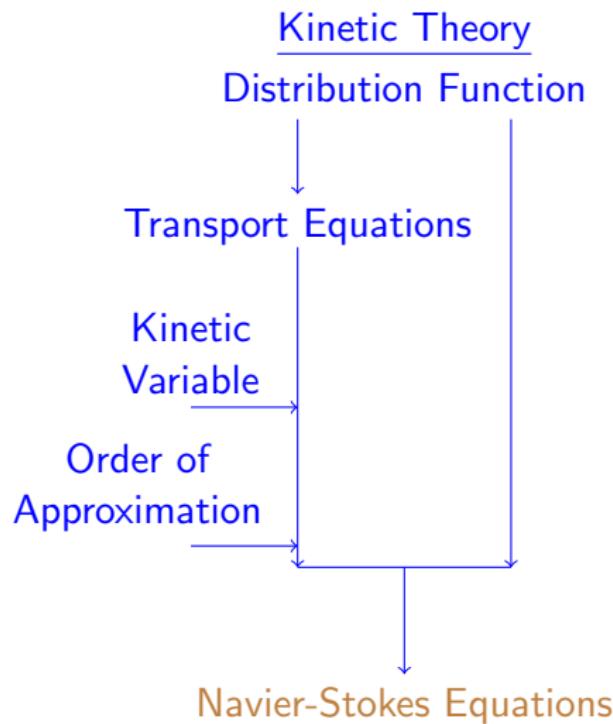


Source: James Chen, University at Buffalo

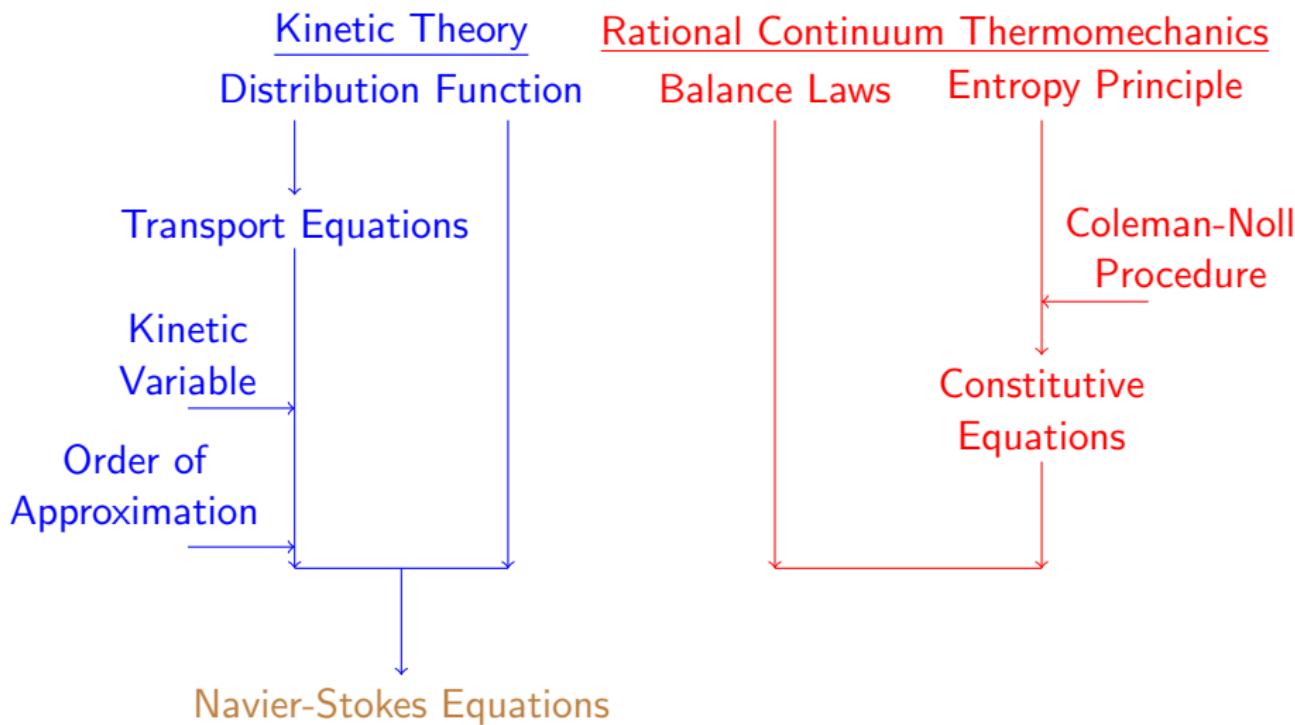
Continuum Theory for Flow with Inner Structures ?

- ➊ Real fluids - Polyatomic molecules
- ➋ High Mach or rarefied flow - Rotation and/or vibration nonequilibrium
- ➌ Turbulence - Eddies

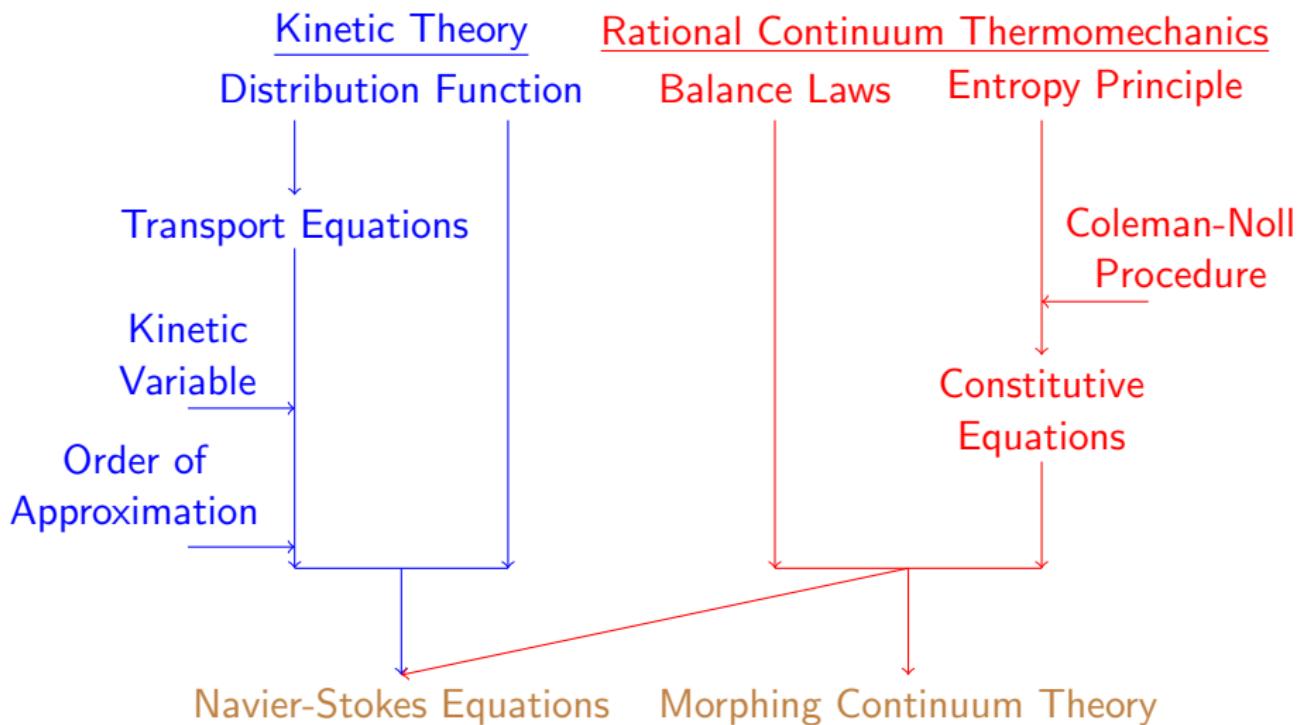
# Roadmap to Continuum Theory



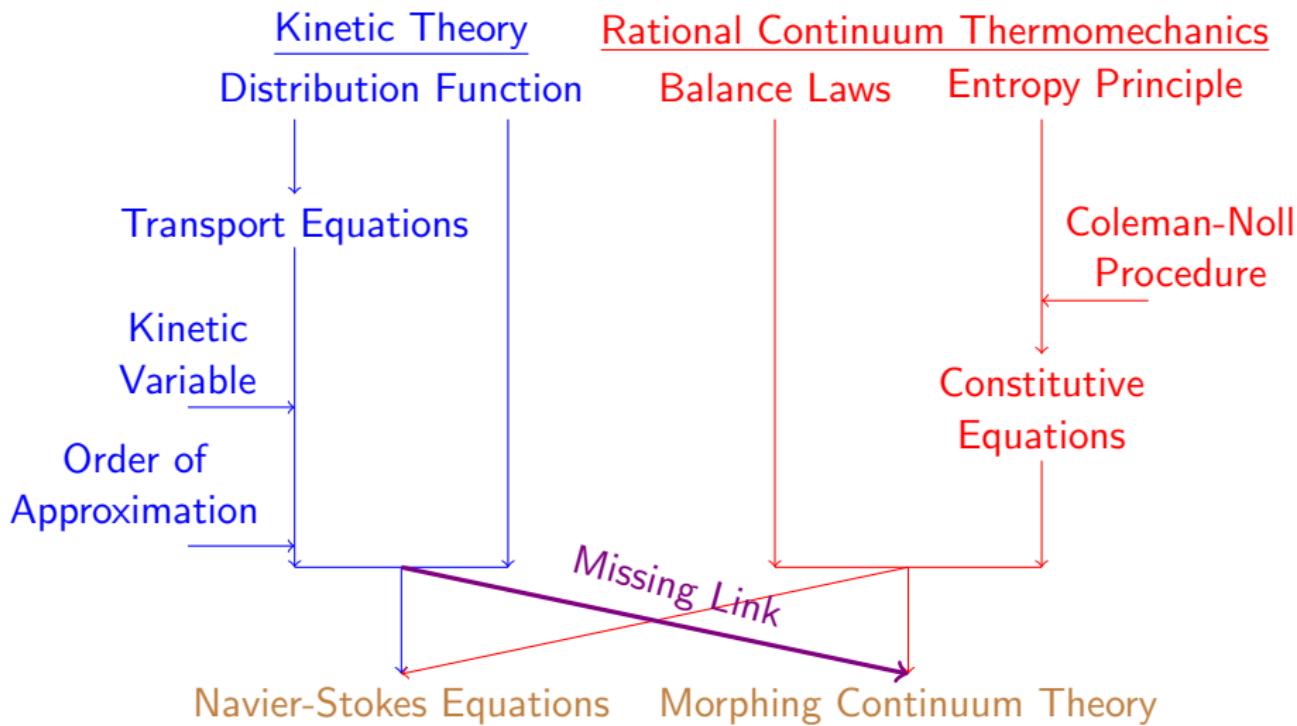
# Roadmap to Continuum Theory



# Roadmap to Continuum Theory



# Roadmap to Continuum Theory



# Advanced Kinetic Theory - Boltzmann-Curtiss Equations

$$\bar{f}(\mathbf{x}, \mathbf{p}, \phi, \mathbf{M}, t) =$$

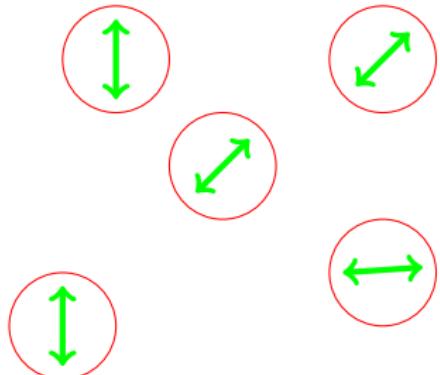
$$\int f(\mathbf{x}, \mathbf{p}, \phi, \mathbf{M}, E_{\text{vib}}, t) dE_{\text{vib}} d\tau,$$

$$(\frac{\partial}{\partial t} + \frac{\mathbf{p}}{m} \frac{\partial}{\partial \mathbf{x}} + \frac{\mathbf{M}}{I} \frac{\partial}{\partial \phi}) \bar{f} = \sum_{\beta} \mathbf{Z}_{\beta}$$

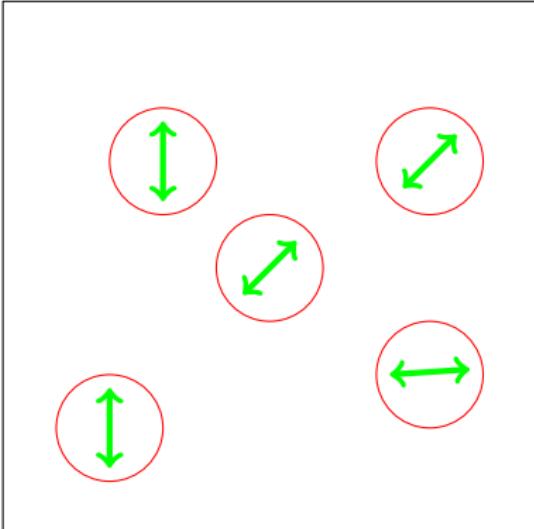
$$\frac{\partial}{\partial t} n\chi + \frac{\partial}{\partial x_i} n\chi v_i - nv_i \frac{\partial \chi}{\partial x_i}$$

$$+ \frac{\partial}{\partial \phi_i} n\chi \omega_i - n\omega_i \frac{\partial \chi}{\partial \phi_i} = 0$$

(Curtiss, 1992)



# Advanced Kinetic Theory - Boltzmann-Curtiss Equations



$$\begin{aligned}\bar{f}(\mathbf{x}, \mathbf{p}, \phi, \mathbf{M}, t) = \\ \int f(\mathbf{x}, \mathbf{p}, \phi, \mathbf{M}, E_{\text{vib}}, t) dE_{\text{vib}} d\tau, \\ (\frac{\partial}{\partial t} + \frac{\mathbf{p}}{m} \frac{\partial}{\partial \mathbf{x}} + \frac{\mathbf{M}}{I} \frac{\partial}{\partial \phi}) \bar{f} = \sum_{\beta} \mathbf{Z}_{\beta} \\ \frac{\partial}{\partial t} n\chi + \frac{\partial}{\partial x_i} n\chi v_i - nv_i \frac{\partial \chi}{\partial x_i} \\ + \frac{\partial}{\partial \phi_i} n\chi \omega_i - n\omega_i \frac{\partial \chi}{\partial \phi_i} = 0\end{aligned}$$

(Curtiss, 1992)

$$\begin{aligned}\bar{f}(\mathbf{x}, \Delta \mathbf{v}, \Delta \boldsymbol{\omega}) = \\ \left( \frac{\sqrt{mI}}{2\pi k\theta} \right)^3 \exp \left[ - \frac{m\Delta \mathbf{v}^2 + I\Delta \boldsymbol{\omega}^2}{2k\theta} \right]\end{aligned}$$

## Boltzmann-Curtiss Distribution

$$\bar{f}(x, \Delta v, \Delta \omega) = \left( \frac{\sqrt{mI}}{2\pi k\theta} \right)^3 \exp \left[ -\frac{m\Delta v^2 + I\Delta \omega^2}{2k\theta} \right]$$

## Boltzmann's H-Theorem

$$H(t) = \int_0^\infty f(E, t) [\log(\frac{f(E, t)}{\sqrt{E}}) - 1] dE$$

# Boltzmann-Curtiss Distribution

$$\bar{f}(x, \Delta v, \Delta \omega) = \left( \frac{\sqrt{mI}}{2\pi k\theta} \right)^3 \exp \left[ -\frac{m\Delta v^2 + I\Delta \omega^2}{2k\theta} \right]$$

## Boltzmann's H-Theorem

$$H(t) = \int_0^\infty f(E, t) [\log(\frac{f(E, t)}{\sqrt{E}}) - 1] dE$$

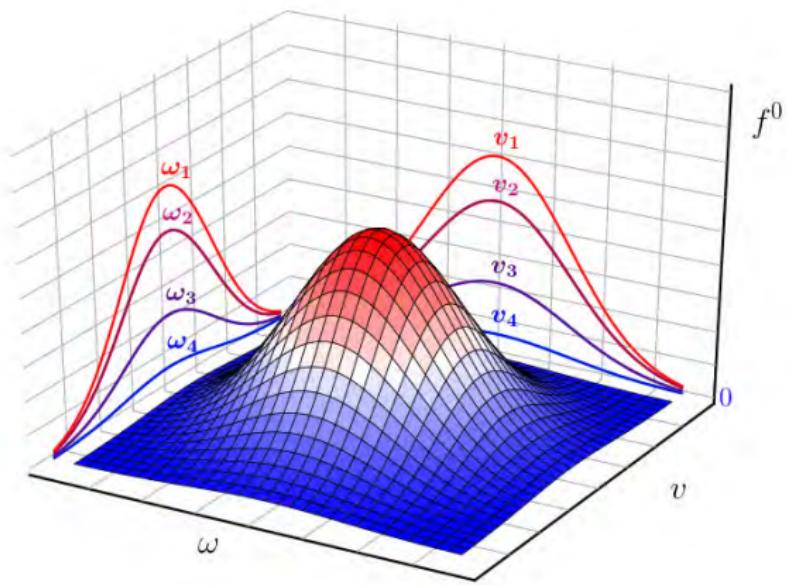
## Method of the Most Probable Distribution (Schrödinger, 1989)

$$W = \frac{N!}{\prod_{i=0}^m n_i!} \quad S = k \ln W$$

Chen, 2017; Wonnell and Chen, 2019

# Maxwell-Boltzmann vs. Boltzmann-Curtiss Distribution

$$\bar{f}(x, \Delta v, \Delta \omega) = \left( \frac{\sqrt{mI}}{2\pi k\theta} \right)^3 \exp \left[ -\frac{m\Delta v^2 + I\Delta \omega^2}{2k\theta} \right]$$



Source: <https://doi.org/10.1115/1.4045761>

# From Kinetic Variables to Transport Equations

## B-C Equation

$$\left( \frac{\partial}{\partial t} + \frac{\mathbf{p}}{m} \frac{\partial}{\partial \mathbf{x}} + \frac{\mathbf{M}}{I} \frac{\partial}{\partial \phi} \right) \bar{f} = \sum_{\beta} \mathbf{Z}_{\beta},$$

## Kinetic Variables

- ① Mass
- ② Linear Momentum
- ③ Angular Momentum
- ④ Internal Energy

## B-C Distribution and Transport Equations

$$\bar{f}(\mathbf{x}, \Delta \mathbf{v}, \Delta \boldsymbol{\omega}) = \left( \frac{\sqrt{ml}}{2\pi k\theta} \right)^3 \exp \left[ -\frac{m\Delta \mathbf{v}^2 + I\Delta \boldsymbol{\omega}^2}{2k\theta} \right]$$

$$\frac{\partial}{\partial t} n\chi + \frac{\partial}{\partial x_i} n\chi v_i - n v_i \frac{\partial \chi}{\partial x_i} = 0$$

# Kinetic Variables in Advanced Kinetic Theory

Mass,  $\chi_1 = m$

$$\frac{\partial}{\partial t} \rho + \frac{\partial}{\partial x_i} (\rho U_i) = 0$$

Linear Momentum,  $\chi_2 = mv_i^* = m(v_i + e_{ijk}\omega_j r_k)$

$$\frac{\partial}{\partial t} (\rho U_j) + \frac{\partial}{\partial x_i} (\rho U_i U_j) = -\frac{\partial}{\partial x_i} (\rho \langle v'_i v'_j \rangle) + \rho e_{jmn} r_n \langle v'_i \omega'_m \rangle$$

Angular Momentum,  $\chi_3 = mr_j r_m \omega_m$

$$\frac{\partial}{\partial t} (\rho i_{jm} W_m) + \frac{\partial}{\partial x_i} (\rho i_{jm} W_m U_i) = -\frac{\partial}{\partial x_i} (\rho i_{jm} \langle \omega'_m v'_i \rangle)$$

Internal Energy,  $\chi_4 = \frac{1}{2}(\langle v'_m v'_m \rangle + \langle r_m r_n \omega'_m \omega'_n \rangle)$

$$\begin{aligned} & \frac{\partial}{\partial t} (\rho e) + \frac{\partial}{\partial x_i} (\rho e U_i) \\ &= -\frac{\partial}{\partial x_i} \left( \frac{1}{2} \rho \left( \langle v'_m v'_m v'_i \rangle + i_{mn} \langle \omega'_m \omega'_n v'_i \rangle \right) \right) + \rho \langle v_i \frac{\partial e}{\partial x_i} \rangle \end{aligned}$$

# First-order Approximation

B-C Equation

BGK Approach

$$\left( \frac{\partial}{\partial t} + \frac{\mathbf{p}}{m} \frac{\partial}{\partial \mathbf{x}} + \frac{\mathbf{M}}{I} \frac{\partial}{\partial \phi} \right) \bar{f} = \left( \frac{\partial \bar{f}}{\partial t} \right)_{\text{collision}},$$

$$\left( \frac{\partial \bar{f}}{\partial t} \right)_{\text{collision}} \Rightarrow -\frac{\bar{f} - \bar{f}^0}{\tau} = -\frac{g}{\tau}$$

Comparison with Morphing Continuum Theory

The classical Boltzmann's equation with a distribution linearly deviates from the Boltzmann distribution (first-order approximation), leads to the celebrated Navier-Stokes equations. Will the B-C equation with a distribution linearly deviated from the B-C distribution give the MCT governing equations?

Wonell and Chen, 2019

# Linear Momentum

## Morphing Continuum Theory

$$\begin{aligned}\frac{\partial}{\partial t}(\rho v_j^{\text{MCT}}) + \frac{\partial}{\partial x_i}(\rho v_i^{\text{MCT}} v_j^{\text{MCT}}) &= -\frac{\partial p}{\partial x_j} + (\lambda + \mu) \frac{\partial^2 v_i^{\text{MCT}}}{\partial x_i \partial x_j} \\ &\quad + (\mu + \kappa) \frac{\partial^2 v_j^{\text{MCT}}}{\partial x_i \partial x_i} + \kappa e_{jlm} \frac{\partial \omega_m^{\text{MCT}}}{\partial x_l}\end{aligned}$$

## Advanced Kinetic Theory

$$\begin{aligned}\frac{\partial}{\partial t}(\rho U_j) + \frac{\partial}{\partial x_i}(\rho U_i U_j) &= -\frac{\partial p}{\partial x_j} + \frac{2n\tau\theta}{3} \frac{\partial^2 U_i}{\partial x_i \partial x_j} \\ &\quad + n\tau\theta \frac{\partial^2 U_j}{\partial x_i \partial x_i} + n\tau\theta e_{jlm} \frac{\partial W_m}{\partial x_l}\end{aligned}$$

# Angular Momentum

## Morphing Continuum Theory

$$\begin{aligned}\frac{\partial}{\partial t}(\rho j \omega_j^{\text{MCT}}) + \frac{\partial}{\partial x_i}(\rho j v_i^{\text{MCT}} \omega_j^{\text{MCT}}) &= (\alpha + \beta) \frac{\partial^2 \omega_i^{\text{MCT}}}{\partial x_i \partial x_j} + \gamma \frac{\partial^2 \omega_j^{\text{MCT}}}{\partial x_i \partial x_i} \\ &\quad + \kappa(e_{jlm} \frac{\partial v_m^{\text{MCT}}}{\partial x_l} - 2\omega_j^{\text{MCT}})\end{aligned}$$

## Advanced Kinetic Theory

$$\frac{\partial}{\partial t}(\rho j W_j) + \frac{\partial}{\partial x_i}(\rho j U_i W_j) = n j \tau \theta \frac{\partial^2 W_j}{\partial x_i \partial x_i} + \nu_r(e_{jlm} \frac{\partial U_m}{\partial x_l} - 2W_j)$$

# Pathway to Navier-Stokes Equations

$$\begin{aligned}\frac{\partial}{\partial t}(\rho U_j) + \frac{\partial}{\partial x_i}(\rho U_i U_j) = & -\frac{\partial p}{\partial x_j} + \frac{2n\tau\theta}{3} \frac{\partial^2 U_i}{\partial x_i \partial x_j} \\ & + n\tau\theta \frac{\partial^2 U_j}{\partial x_i \partial x_i} + n\tau\theta e_{jlm} \frac{\partial W_m}{\partial x_l}\end{aligned}$$

# Pathway to Navier-Stokes Equations

$$\begin{aligned}\frac{\partial}{\partial t}(\rho U_j) + \frac{\partial}{\partial x_i}(\rho U_i U_j) = & -\frac{\partial p}{\partial x_j} + \frac{2n\tau\theta}{3} \frac{\partial^2 U_i}{\partial x_i \partial x_j} \\ & + n\tau\theta \frac{\partial^2 U_j}{\partial x_i \partial x_i} + n\tau\theta e_{jlm} \frac{\partial W_m}{\partial x_l}\end{aligned}$$

## Angular Motion Equivalence

$$\vec{W} = \frac{1}{2} \nabla \times \vec{U}$$

# Pathway to Navier-Stokes Equations

$$\begin{aligned}\frac{\partial}{\partial t}(\rho U_j) + \frac{\partial}{\partial x_i}(\rho U_i U_j) = & -\frac{\partial p}{\partial x_j} + \frac{2n\tau\theta}{3} \frac{\partial^2 U_i}{\partial x_i \partial x_j} \\ & + n\tau\theta \frac{\partial^2 U_j}{\partial x_i \partial x_i} + n\tau\theta e_{jlm} \frac{\partial W_m}{\partial x_l}\end{aligned}$$

## Angular Motion Equivalence

$$\vec{W} = \frac{1}{2} \nabla \times \vec{U}$$

## Navier-Stokes Equations Type II

$$\frac{\partial}{\partial t}(\rho U_j) + \frac{\partial}{\partial x_i}(\rho U_i U_j) = -\frac{\partial p}{\partial x_j} + \frac{7n\tau\theta}{6} \frac{\partial^2 U_i}{\partial x_i \partial x_j} + \frac{n\tau\theta}{2} \frac{\partial^2 U_j}{\partial x_i \partial x_i}$$

# Navier-Stokes Equations Type I & II

## Navier-Stokes Equations Type II - reduced from MCT

$$\frac{\partial}{\partial t}(\rho U_j) + \frac{\partial}{\partial x_i}(\rho U_i U_j) = -\frac{\partial p}{\partial x_j} + \frac{7n\tau\theta}{6}\frac{\partial^2 U_i}{\partial x_i \partial x_j} + \frac{n\tau\theta}{2}\frac{\partial^2 U_j}{\partial x_i \partial x_i}$$

## Navier-Stokes Equations Type I - classical framework

$$\frac{\partial}{\partial t}(\rho U_j) + \frac{\partial}{\partial x_i}(\rho U_i U_j) = -\frac{\partial p}{\partial x_j} + \frac{n\tau\theta}{3}\frac{\partial^2 U_i}{\partial x_i \partial x_j} + n\tau\theta\frac{\partial^2 U_j}{\partial x_i \partial x_i}$$

# A Map between Kinetic Theory and Continuum Theory

Boltzmann-Curtiss Equation

$$\frac{\partial}{\partial \phi_i} = 0$$

No angular dependence

Boltzmann Equation

# A Map between Kinetic Theory and Continuum Theory

Boltzmann-Curtiss Equation

$$\frac{\partial}{\partial \phi_i} = 0$$

No angular dependence

Boltzmann Equation

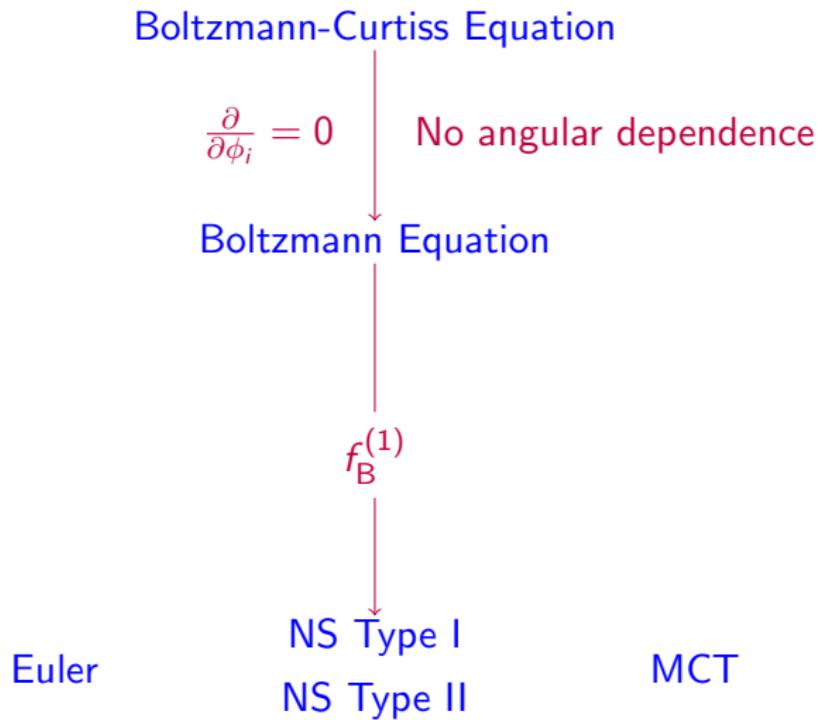
Euler

NS Type I

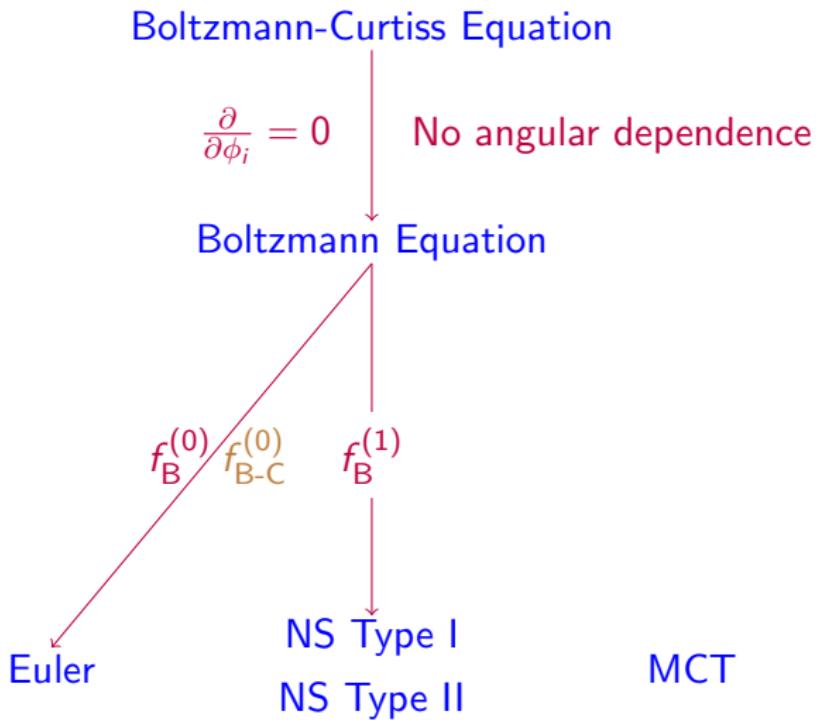
NS Type II

MCT

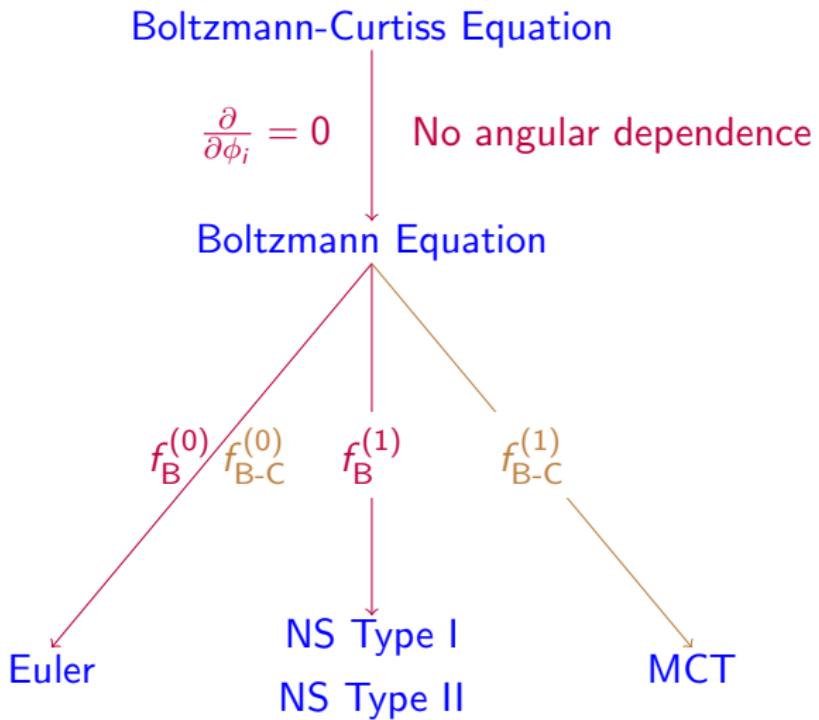
# A Map between Kinetic Theory and Continuum Theory



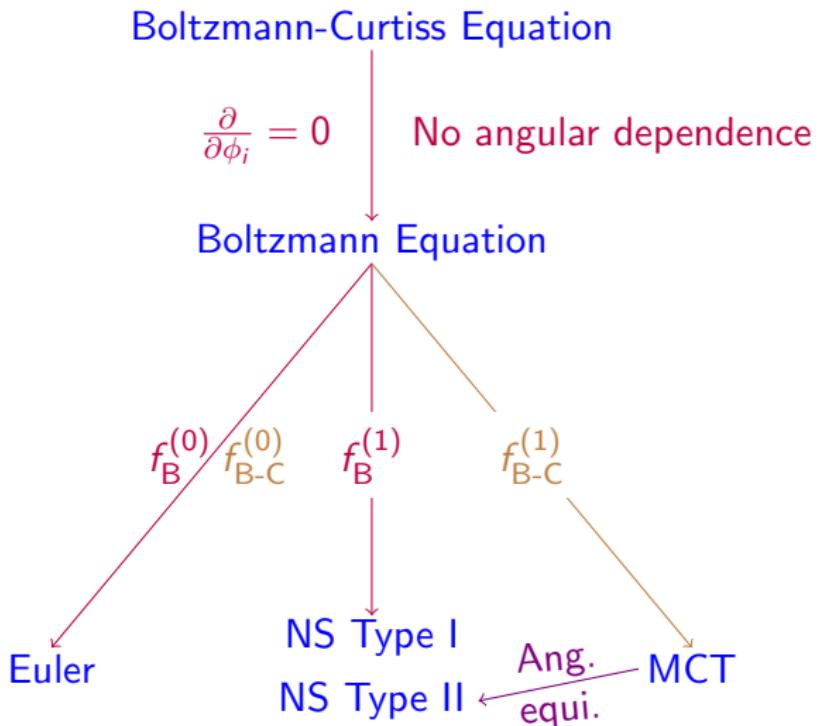
# A Map between Kinetic Theory and Continuum Theory



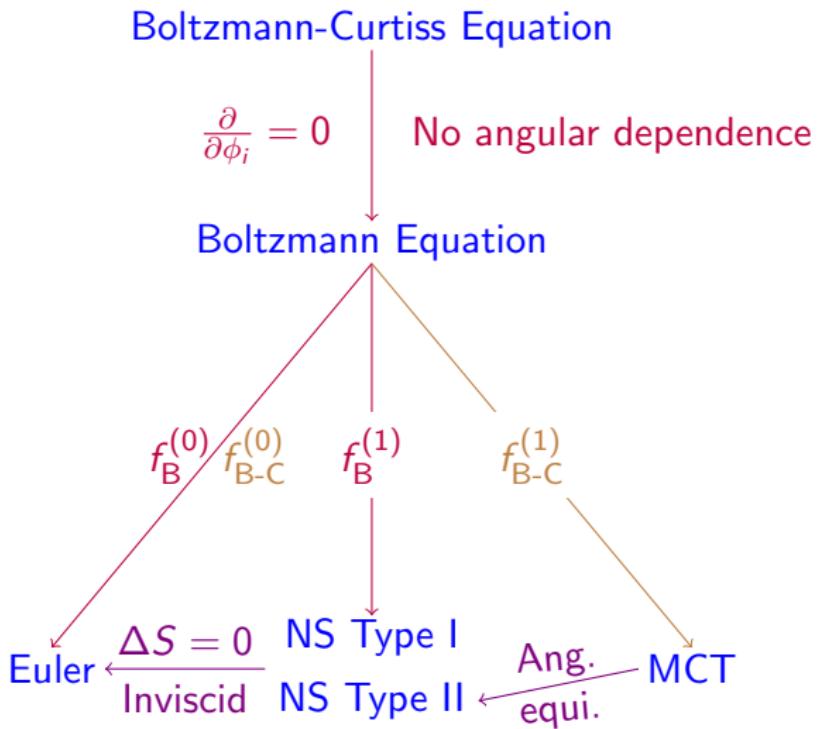
# A Map between Kinetic Theory and Continuum Theory



# A Map between Kinetic Theory and Continuum Theory



# A Map between Kinetic Theory and Continuum Theory



# Morphing Continuum Theory for Incompressible Fluids

## Conservation of Mass

$$\frac{\partial}{\partial t}\rho + \frac{\partial}{\partial x_i}(\rho U_i) = 0$$

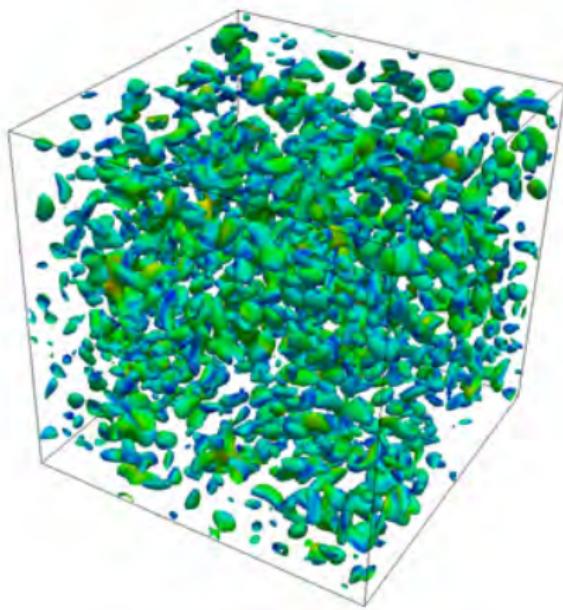
## Balance Law of Linear Momentum

$$\frac{\partial}{\partial t}(U_j) + U_i \frac{\partial}{\partial x_i}(U_j) = \frac{1}{\rho} \frac{\partial p}{\partial x_j} + \frac{\mu+\kappa}{\rho} \frac{\partial^2 U_j}{\partial x_i \partial x_i} + \frac{\kappa}{\rho} e_{jkl} \frac{\partial W_l}{\partial x_k}$$

## Balance Law of Angular Momentum

$$\frac{\partial}{\partial t}(W_m) + U_i \frac{\partial}{\partial x_i}(W_m) = \frac{\gamma}{\rho j} \frac{\partial W_j}{\partial x_i \partial x_i} + \frac{\kappa}{\rho j} (e_{jkl} \frac{\partial U_l}{\partial x_k} - 2W_j)$$

# Homogeneous Isotropic Turbulence



Source: James Chen, University at Buffalo

# Linear Momentum Equations

## Balance Law of Linear Momentum

$$\frac{d}{dt} \langle U_j \rangle = \frac{1}{\rho} \left\langle \frac{\partial p}{\partial x_j} \right\rangle + \frac{\mu + \kappa}{\rho} \left\langle \frac{\partial^2 U_j}{\partial x_i \partial x_i} \right\rangle + \frac{\kappa}{\rho} \left\langle e_{jkl} \frac{\partial U_l}{\partial W_k} \right\rangle$$

$$\frac{d \langle U_j \rangle}{dt} = \cancel{\frac{1}{\rho} \left\langle \frac{\partial p}{\partial x_j} \right\rangle} + \cancel{\frac{\mu + \kappa}{\rho} \left\langle \frac{\partial^2 U_j}{\partial x_i \partial x_i} \right\rangle} + \cancel{\frac{\kappa}{\rho} \left\langle e_{jkl} \frac{\partial W_l}{\partial x_k} \right\rangle}$$

$$\frac{d}{dt} \langle U_j \rangle = 0$$

# Angular Momentum Equations

## Balance Law of Angular Momentum

$$\frac{d}{dt} \langle W_j \rangle = \frac{\gamma}{\rho j} \left\langle \frac{\partial^2 W_j}{\partial x_i \partial x_i} \right\rangle + \frac{\kappa}{\rho j} \left( \left\langle e_{jkl} \frac{\partial U_l}{\partial x_k} \right\rangle - 2 \left\langle W_j \right\rangle \right)$$

$$\frac{d}{dt} \langle W_j \rangle = \cancel{\frac{\gamma}{\rho j} \left\langle \frac{\partial^2 W_j}{\partial x_i \partial x_i} \right\rangle} + \frac{\kappa}{\rho j} \left( \cancel{\left\langle e_{jkl} \frac{\partial U_l}{\partial x_k} \right\rangle} - 2 \left\langle W_j \right\rangle \right)$$

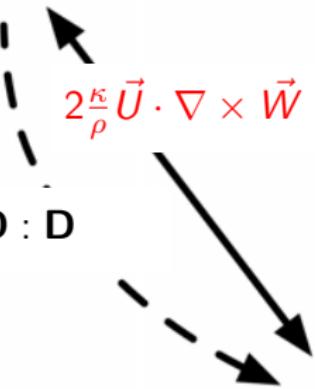
$$\frac{d}{dt} \langle W_j \rangle = -\frac{2\kappa}{\rho j} \left\langle W_j \right\rangle$$

$$\langle W_j \rangle = Ce^{-\tau_w t} \quad \tau_w = \frac{2\kappa}{\rho j}: \text{turbulence relaxation time}$$

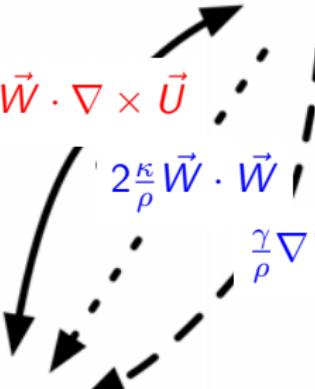
Wonell and Chen, 2019; Chen, 2003 (Science)

# Energy Equation

Translational  
Kinetic  
Energy

$$\frac{\mu + \kappa}{2\rho} \mathbf{D} : \mathbf{D}$$
$$2\frac{\kappa}{\rho} \vec{U} \cdot \nabla \times \vec{W}$$


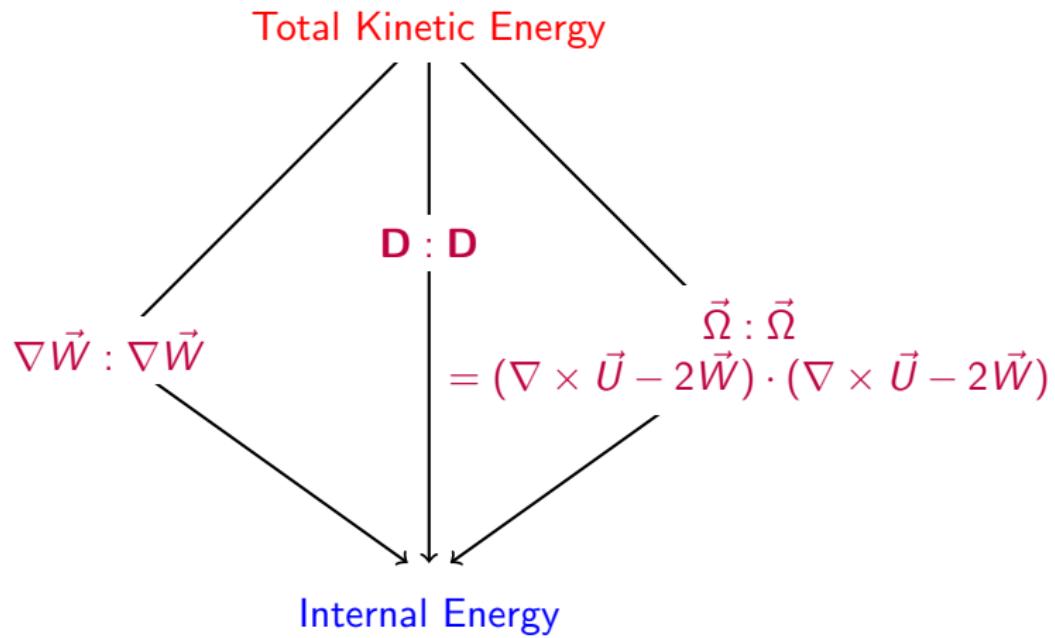
Rotational  
Kinetic  
Energy

$$2\frac{\kappa}{\rho} \vec{W} \cdot \nabla \times \vec{U}$$
$$2\frac{\kappa}{\rho} \vec{W} \cdot \vec{W}$$
$$\frac{\gamma}{\rho} \nabla \vec{W} : \nabla \vec{W}$$


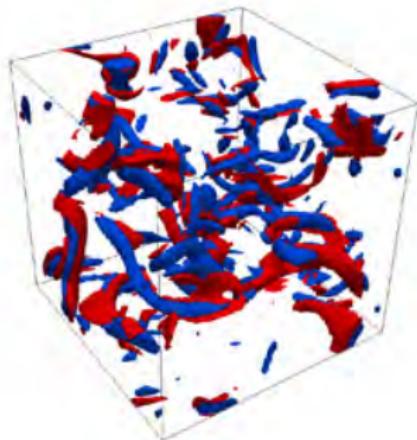
Internal  
Energy

# Total Energy Cascade

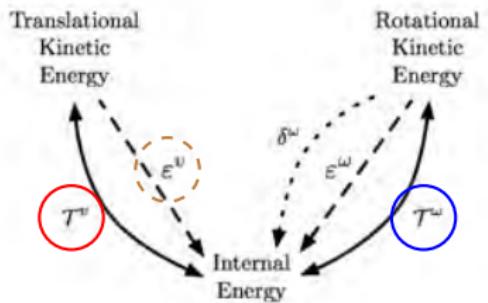
## Energy Transfer & Absolute Rotation



# Homogeneous Isotropic Turbulence - Energy Transfer



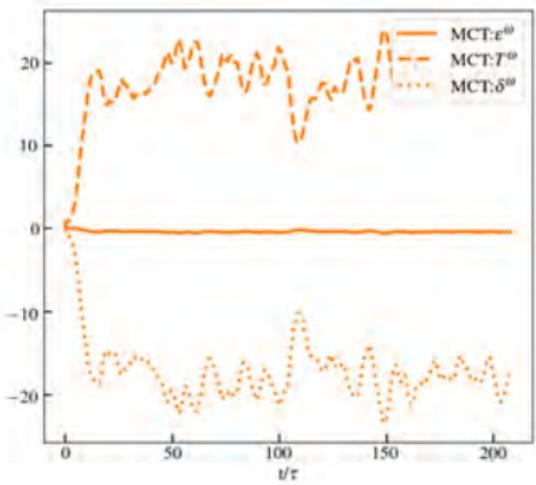
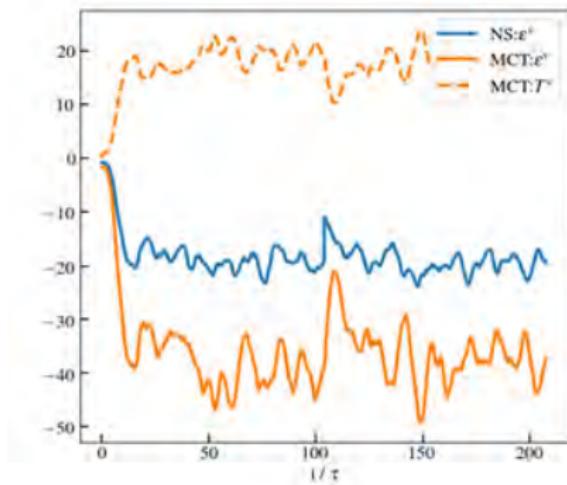
Translational energy cascade



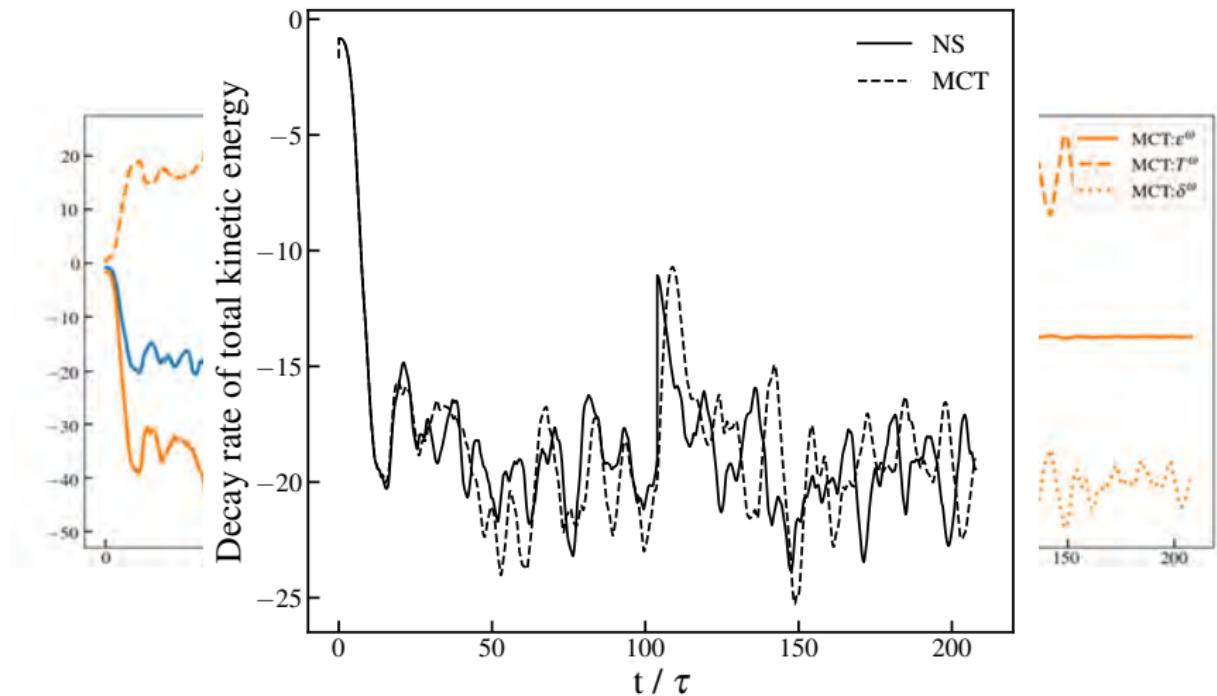
Rotational energy cascade

Source: James Chen, University at Buffalo

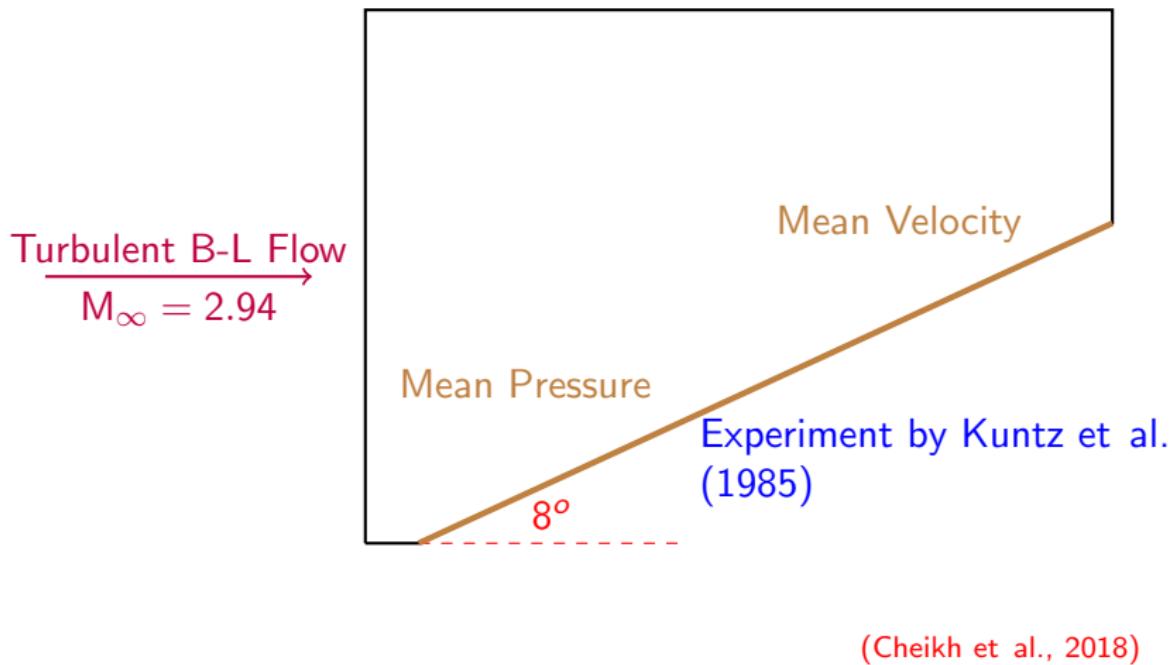
# Homogeneous Isotropic Turbulence - MCT vs. NS



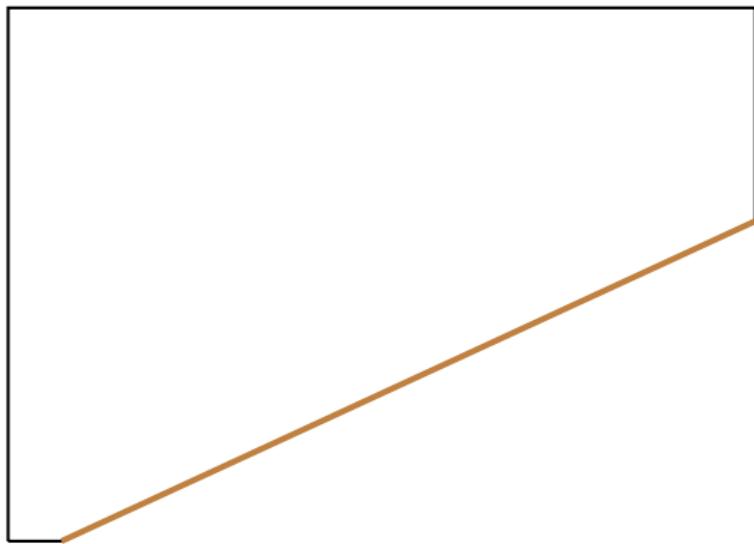
# Homogeneous Isotropic Turbulence - MCT vs. NS



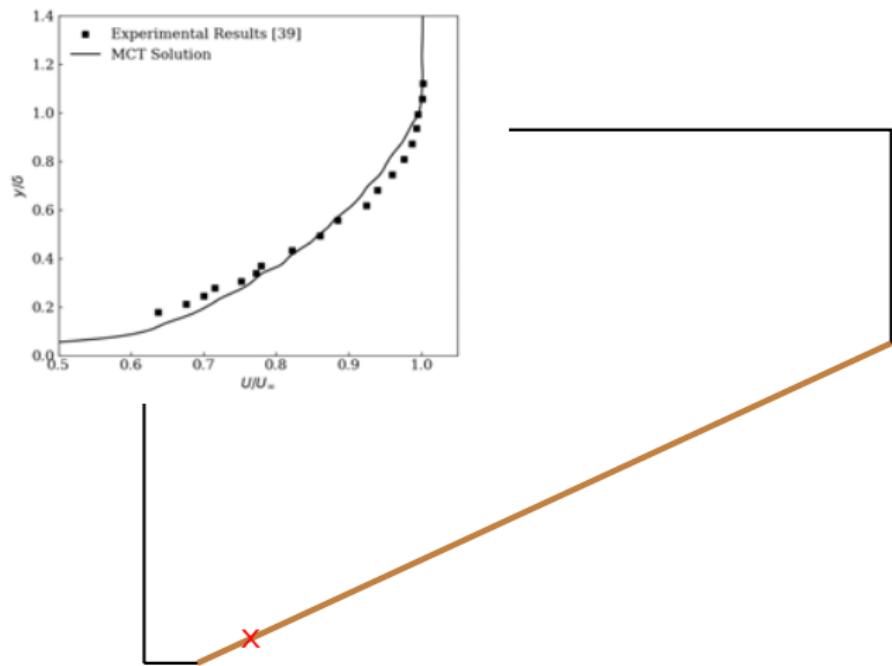
# Supersonic Flow Over an $8^\circ$ Compression Ramp



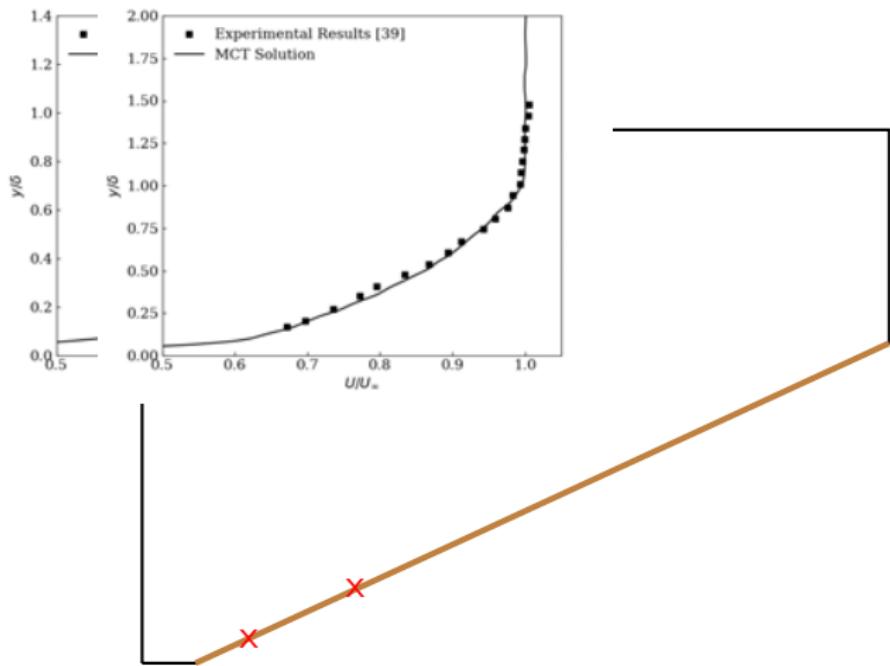
# Supersonic Flow - Compression Ramp



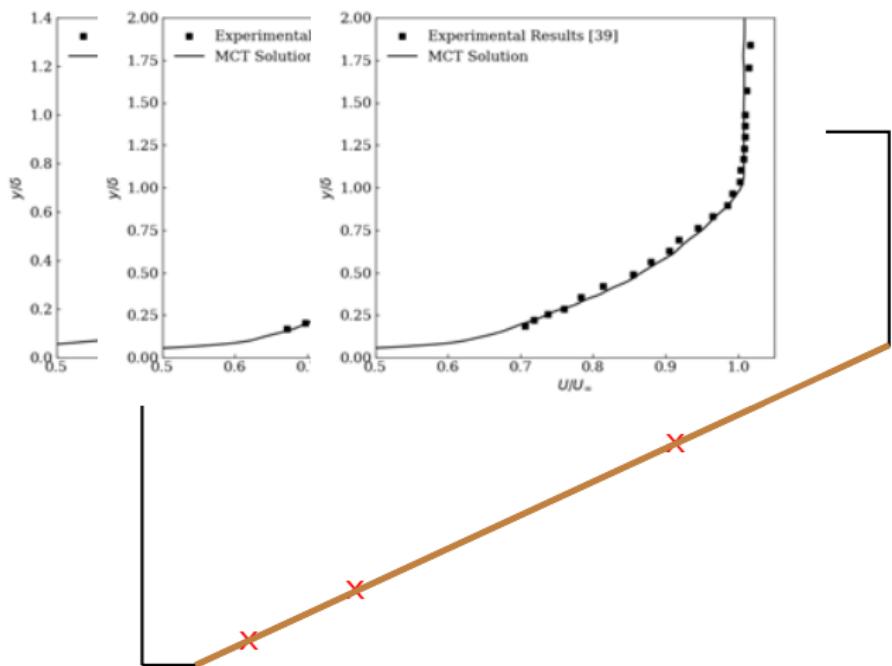
# Supersonic Flow - Compression Ramp



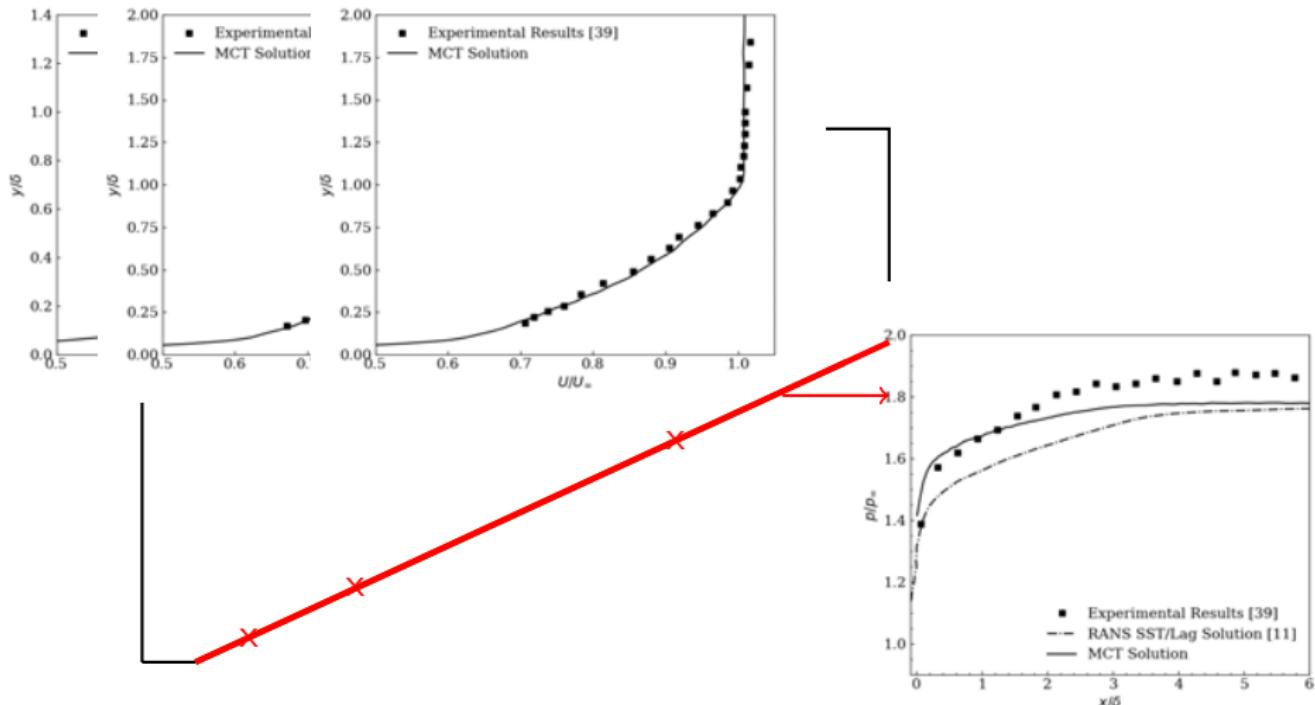
# Supersonic Flow - Compression Ramp



# Supersonic Flow - Compression Ramp

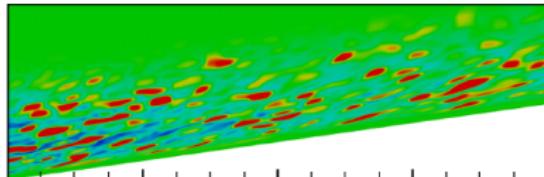


# Supersonic Flow - Compression Ramp

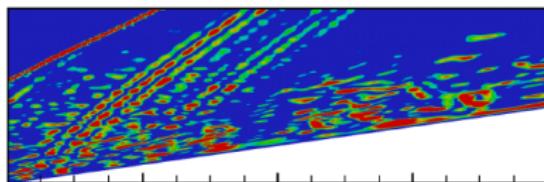


# Supersonic Flow - Energy Analysis

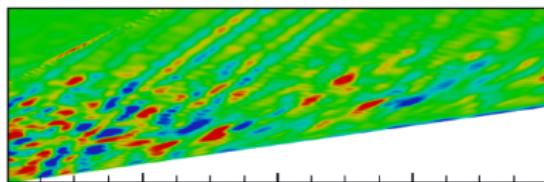
Rotation



Translation



Internal



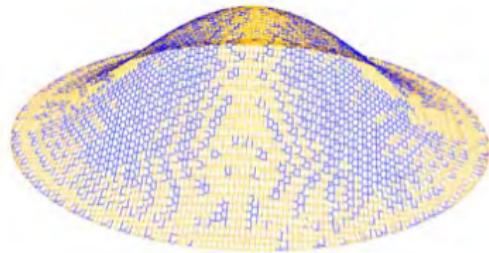
$$\frac{j}{2} \langle \omega_i'' \omega_i'' \rangle \longleftrightarrow \frac{1}{2} \langle v_i'' v_i'' \rangle \longleftrightarrow \langle e'' \rangle$$

# Transonic Flow Over an Axisymmetric Hill

## Flow Profile

$M_\infty$	0.6
$\delta_{\text{inlet}}$	0.039 m
$H_{\text{bump}}$	0.078 m
$Re_H$	6709

## Hill Surface Pressure Coefficient



- Experiment: Simpson et al. (2002)
- NS-based DNS: Castagna et al. (2014)
- MCT-based DNS: Wonnell, Cheikh, and Chen (2018)

From "Morphing Continuum Simulation of Transonic Flow over Axisymmetric Hill" by Louis B. Wonnell, Mohamad I. Cheikh and James Chen; reprinted by permission of the American Institute of Aeronautics and Astronautics, Inc.  
Source: <https://doi.org/10.2514/1.J057064>

# Transonic Flow - Vortex Visualization and Surface Pressure

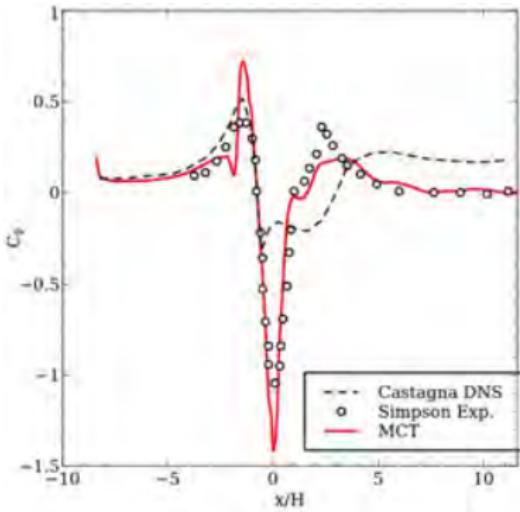
## Objective Q-Criterion for Vortex Visualization



$$2II_a = (\nabla \cdot \vec{v})^2 - \nabla \vec{v} : \nabla \vec{v}^T - 2(\nabla \times \vec{v}) \cdot \vec{\omega} + 2\vec{\omega} \cdot \vec{\omega}$$

Source: James Chen, University at Buffalo

## Hill Surface Pressure Coefficient



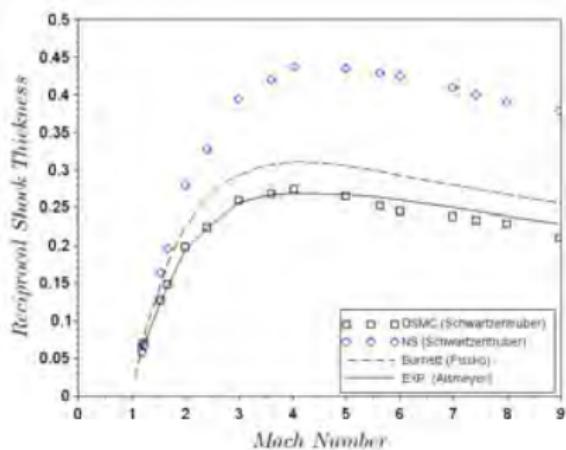
Computation Cost:  
MCT: 6M cells  
DNS: 54M cells

From "Morphing Continuum Simulation of Transonic Flow over Axisymmetric Hill" by Louis R. Womell, Mohamed I. Chedk and James Chen; reprinted by permission of the American Institute of Aeronautics and Astronautics, Inc.  
Source: <https://doi.org/10.2514/1.J057064>

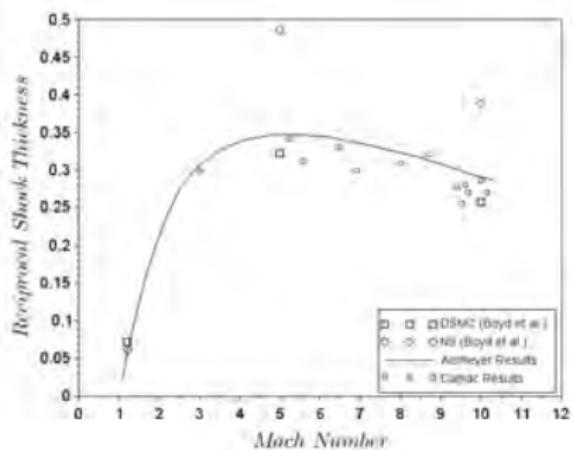
# 1-D Shock Profile – Argon and Nitrogen



Argon



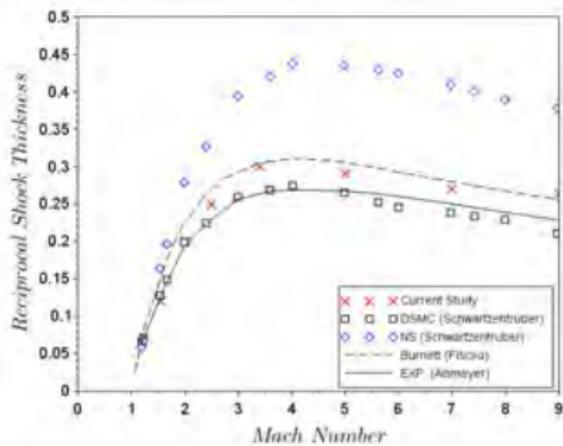
Nitrogen



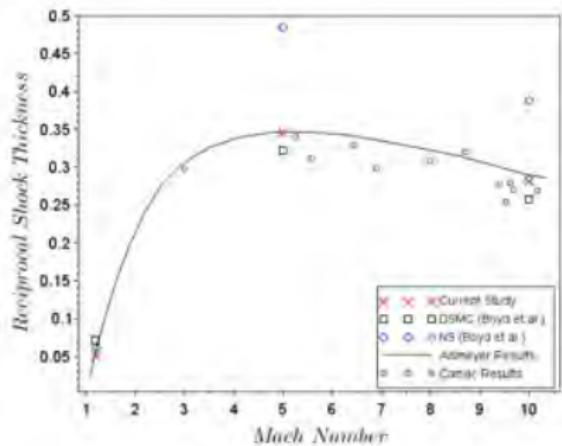
# 1-D Shock Profile – Argon and Nitrogen



Argon

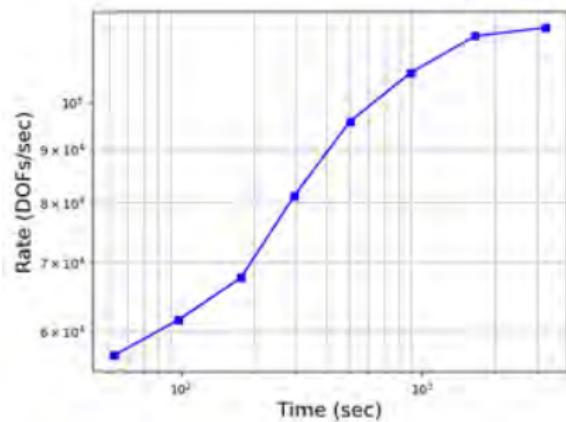
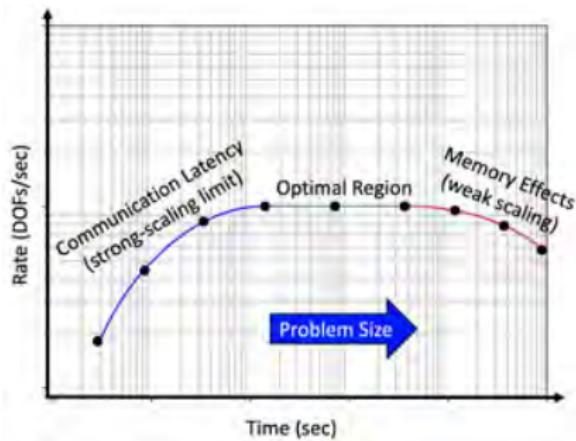


Nitrogen



Source: <https://doi.org/10.1115/1.4045761>

# Scalability - Static Scaling



Sementilli, McGurn, and Chen, 2023

- Number of Cores Tested: 32,768 (DoE Quartz)
- Test Case: Multiphase (liquid and gas) Kelvin-Helmholtz instability
- Degrees of Freedom Tested: 48M

Source: <https://doi.org/10.1002/fld.5169>

# Integrated Computational Materials Engineering for Hypersonics



Source: [https://en.wikipedia.org/wiki/Atmospheric\\_entry#/media/File:Apollo\\_cm.jpg](https://en.wikipedia.org/wiki/Atmospheric_entry#/media/File:Apollo_cm.jpg)

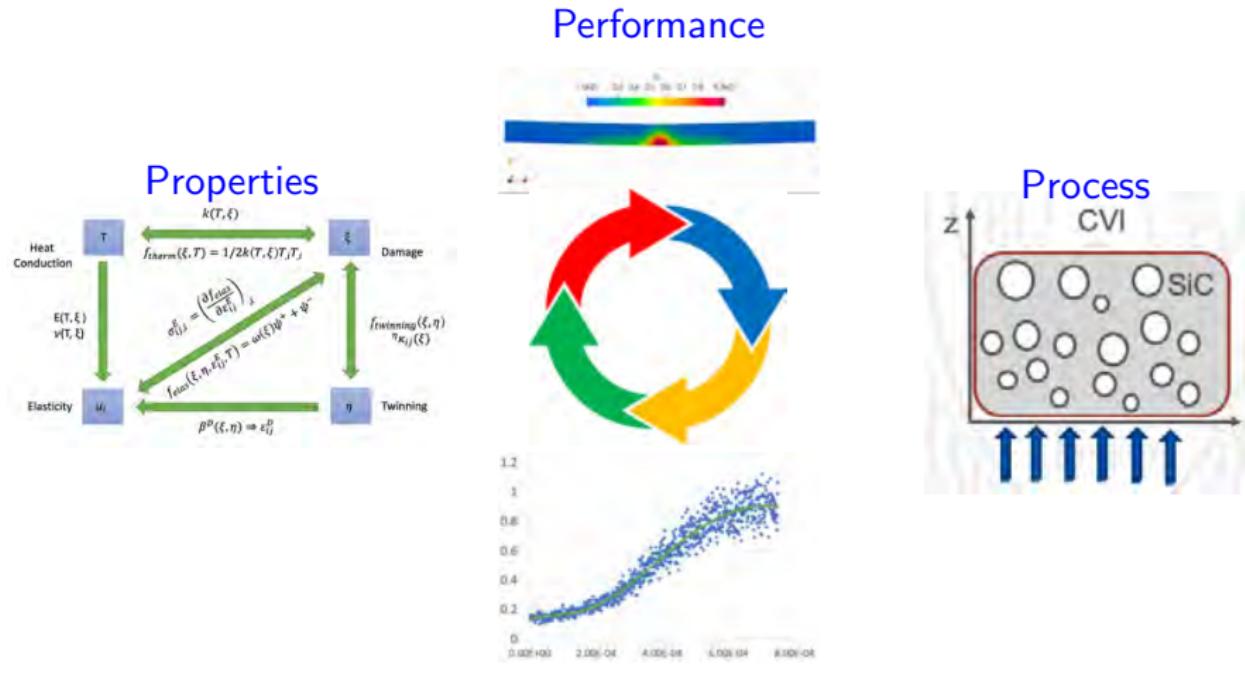
- Limited data availability for ceramics under hypersonic condition
  - Bayesian-based uncertainty management

- Insufficient understanding at vehicle/flow interface
  - Artificial Neural Network with Bayesian statistics
  - Physics-based models with exascale simulation



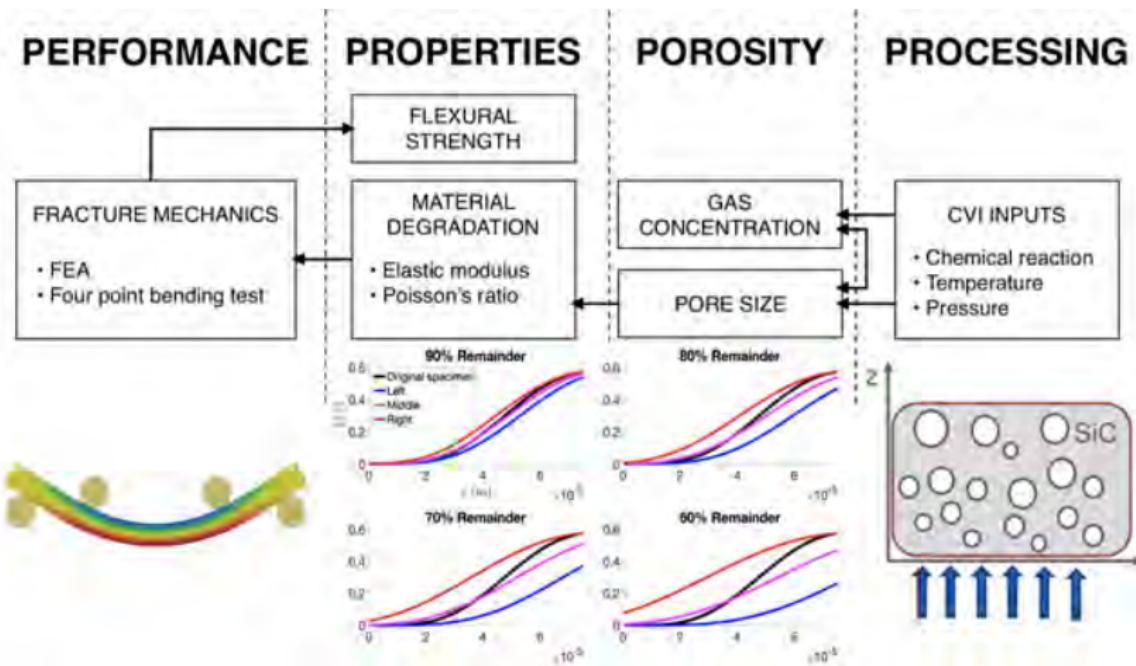
Source: Getty Images

# ICME Framework for Ceramics

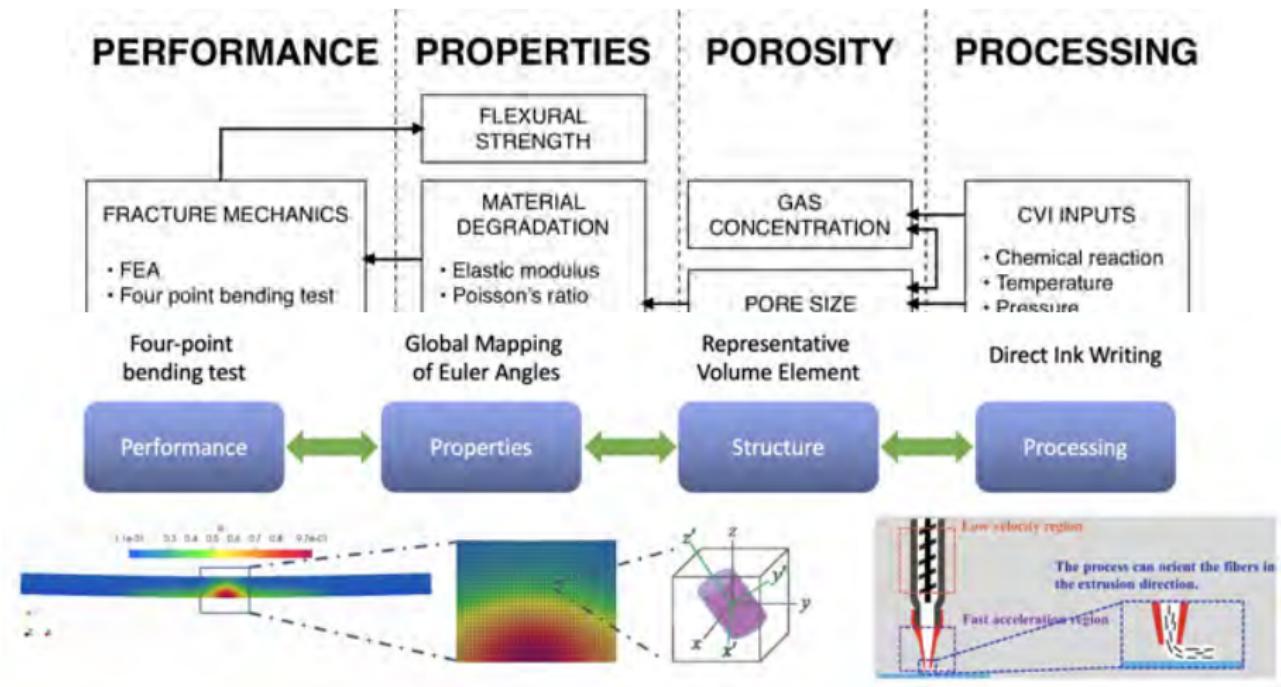


Source: James Chen, University at Buffalo

# Manufacturing-Driven ICME for CMC in Hypersonics



# Manufacturing-Driven ICME for CMC in Hypersonics

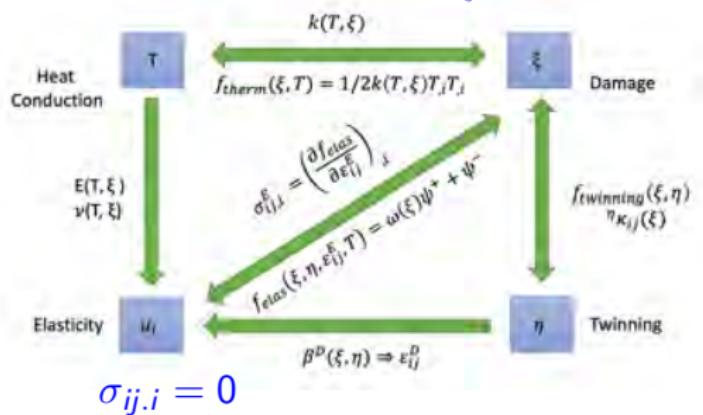


Source: James Chen, University at Buffalo

# Four-Way Thermo-Mechanical Coupling

$$\frac{\partial T}{\partial t} = w(\xi)k(T)T_{,ii}$$

$$\frac{\partial \xi}{\partial t} = -L_\xi \left( \frac{\partial f_{loc}(\xi, \epsilon, T, \eta)}{\partial \xi} - \kappa \xi_{,ii} \right)$$

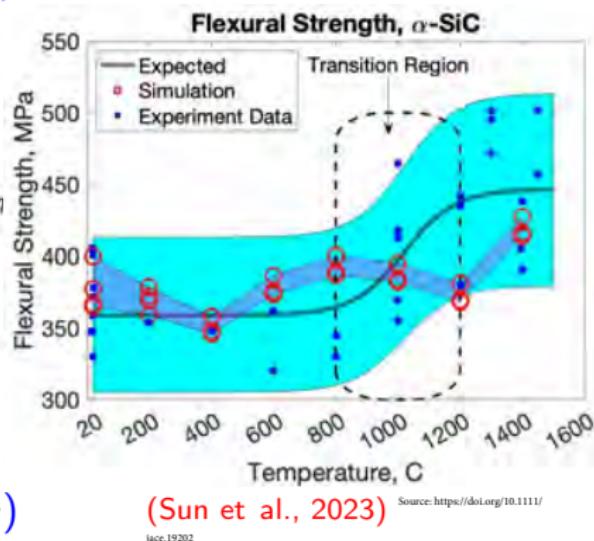
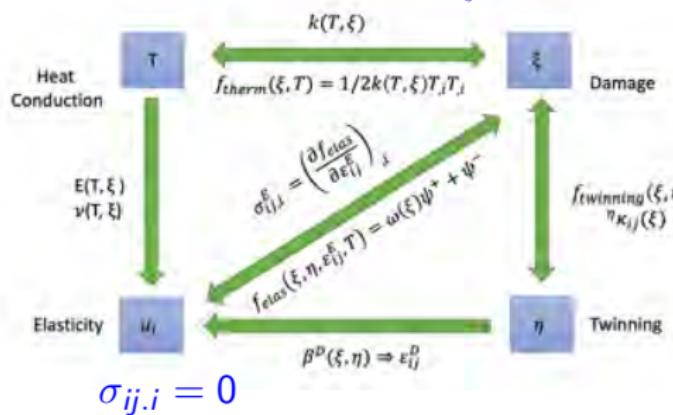


$$\frac{\partial \eta}{\partial t} = -L_\eta \left( \frac{\partial f_{loc}(\xi, \epsilon, T, \eta)}{\partial \eta} - \kappa \eta_{,ii} \right)$$

# Four-Way Thermo-Mechanical Coupling

$$\frac{\partial T}{\partial t} = w(\xi)k(T)T_{,ii}$$

$$\frac{\partial \xi}{\partial t} = -L_\xi \left( \frac{\partial f_{loc}(\xi, \epsilon, T, \eta)}{\partial \xi} - \kappa \xi_{,ii} \right)$$

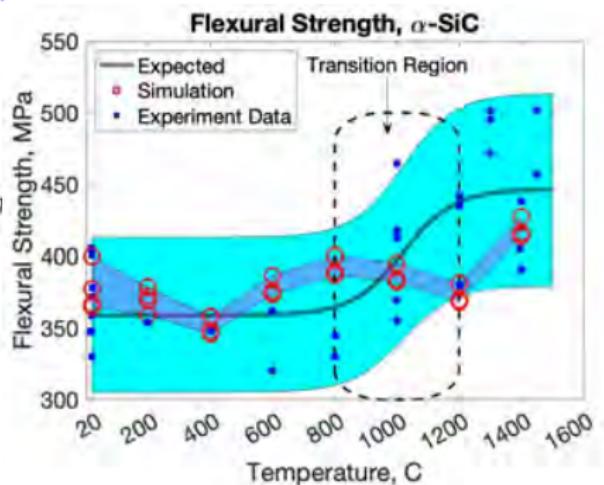
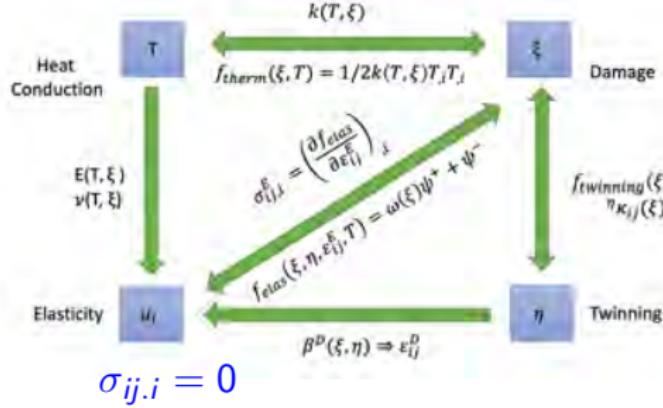


# Four-Way Thermo-Mechanical Coupling

$$\frac{\partial T}{\partial t} = w(\xi)k(T)T_{,ii}$$

$$\frac{\partial \xi}{\partial t} = -L_\xi \left( \frac{\partial f_{loc}(\xi, \epsilon, T, \eta)}{\partial \xi} - \kappa \xi_{,ii} \right)$$

$20 \sim 1400^\circ C$

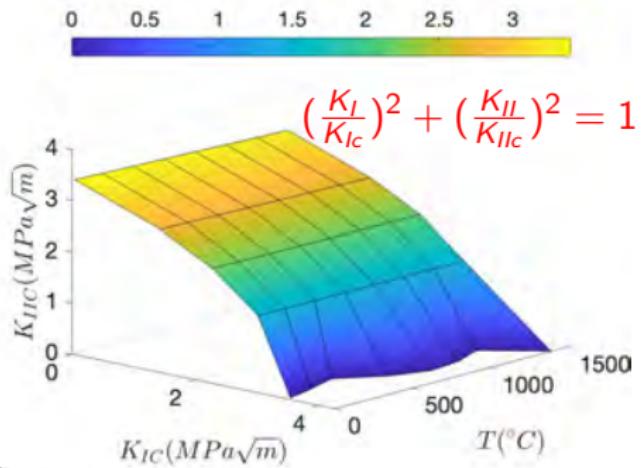
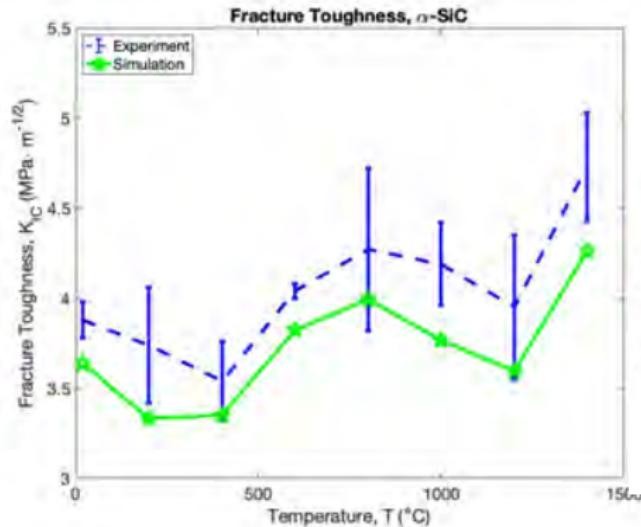


$$\frac{\partial \eta}{\partial t} = -L_\eta \left( \frac{\partial f_{loc}(\xi, \epsilon, T, \eta)}{\partial \eta} - \kappa \eta_{,ii} \right)$$

(Sun et al., 2023)

Source: <https://doi.org/10.1111/jace.19202>

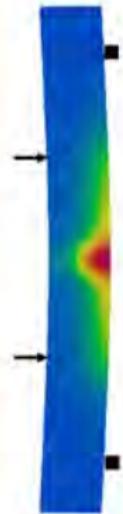
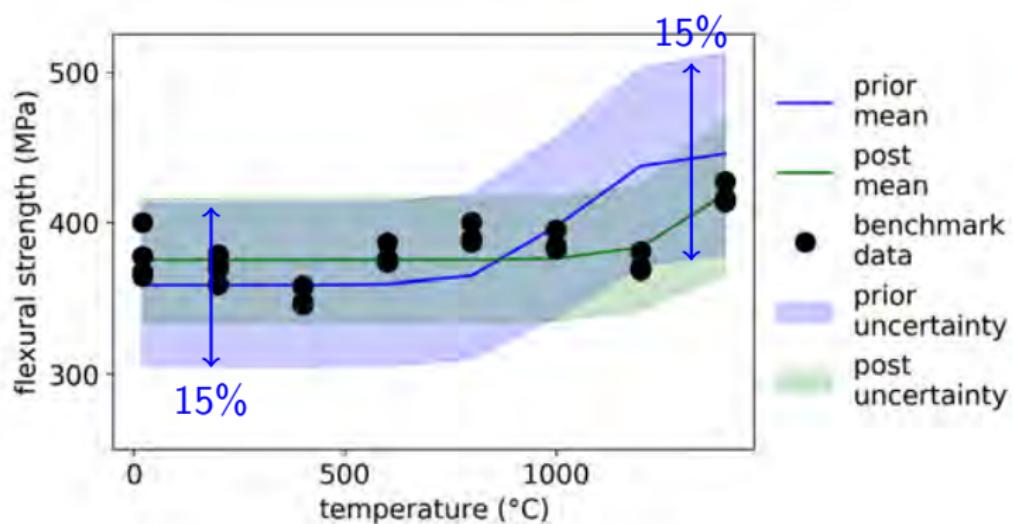
# Damage Criteria of $\alpha$ -SiC



(Sun et al., 2023)

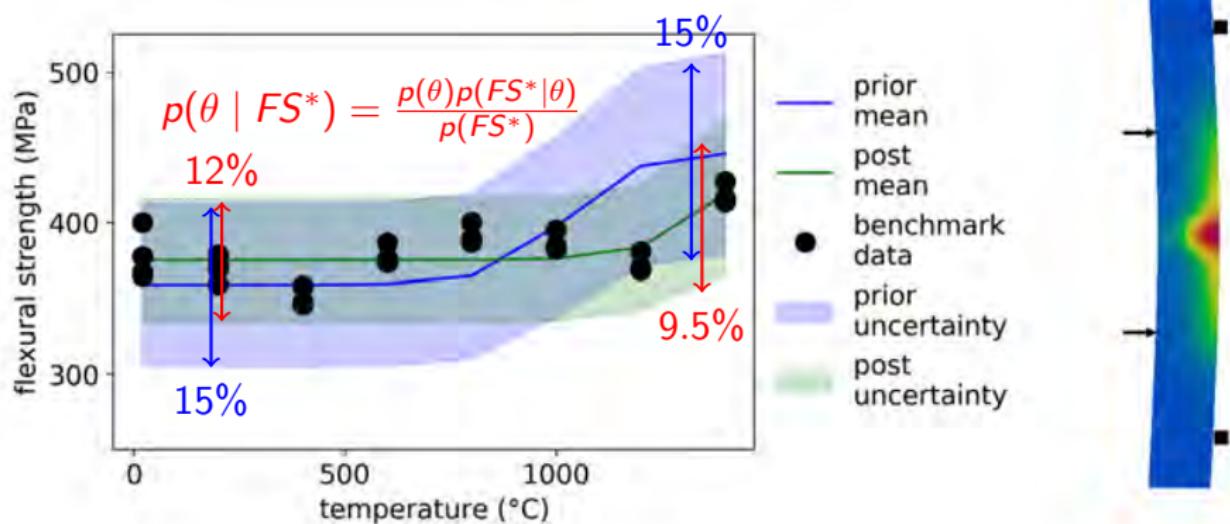
Source: <https://doi.org/10.1111/jace.19202>

# Uncertainty Management with Bayesian Update



Source: James Chen, University at Buffalo

# Uncertainty Management with Bayesian Update

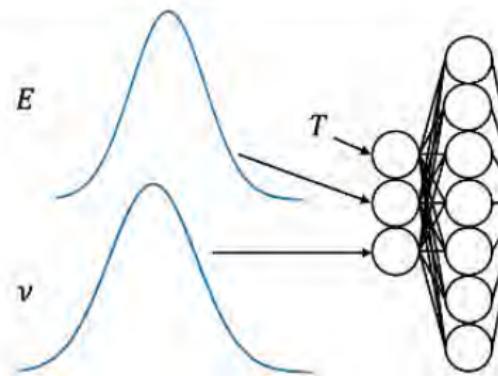


Source: James Chen, University at Buffalo

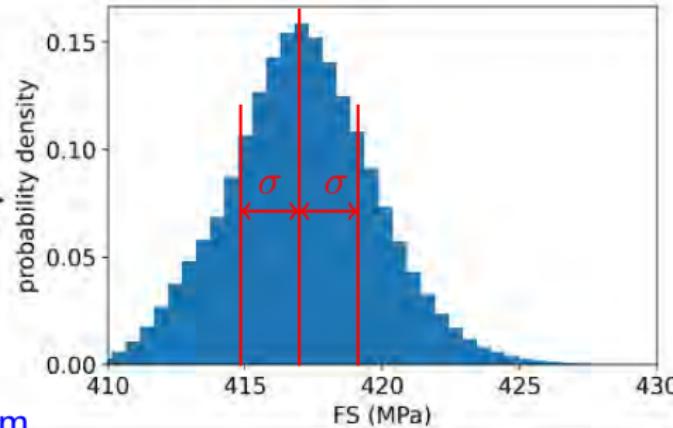
(Walker, Sun, and Chen, under revision)

# Uncertainty-Informed Artificial Neural Network

## Forward Uncertainty Sampling



$\alpha\text{-SiC at } 1400\text{ }^{\circ}\text{C}$



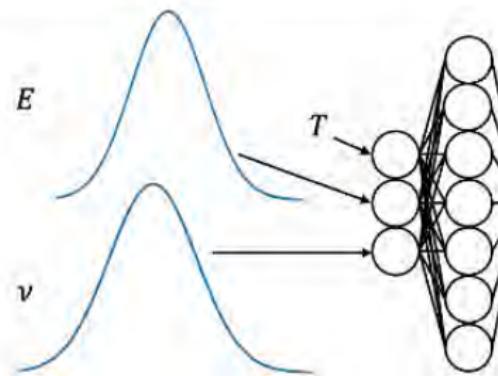
BFGS Algorithm

$$y^k = g^{k+1} - g^k$$

Source: James Chen, University at Buffalo

# Uncertainty-Informed Artificial Neural Network

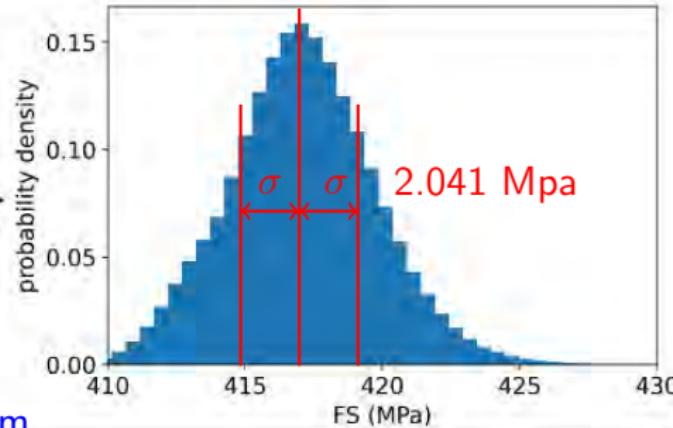
## Forward Uncertainty Sampling



BFGS Algorithm

$$y^k = g^{k+1} - g^k$$

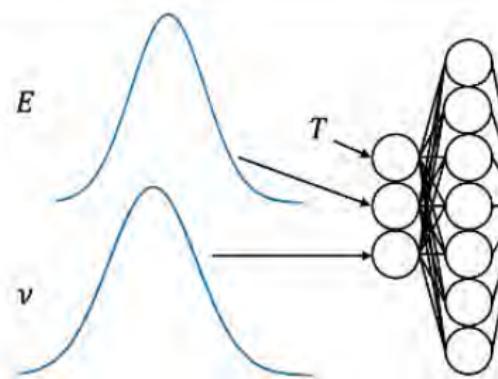
$\alpha\text{-SiC at } 1400 \text{ }^\circ\text{C}$



Source: James Chen, University at Buffalo

# Uncertainty-Informed Artificial Neural Network

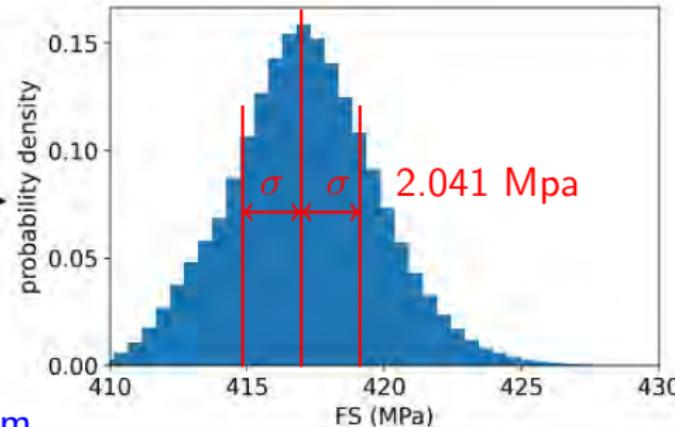
## Forward Uncertainty Sampling



BFGS Algorithm

$$y^k = g^{k+1} - g^k$$

$\alpha\text{-SiC at } 1400\text{ }^\circ\text{C}$



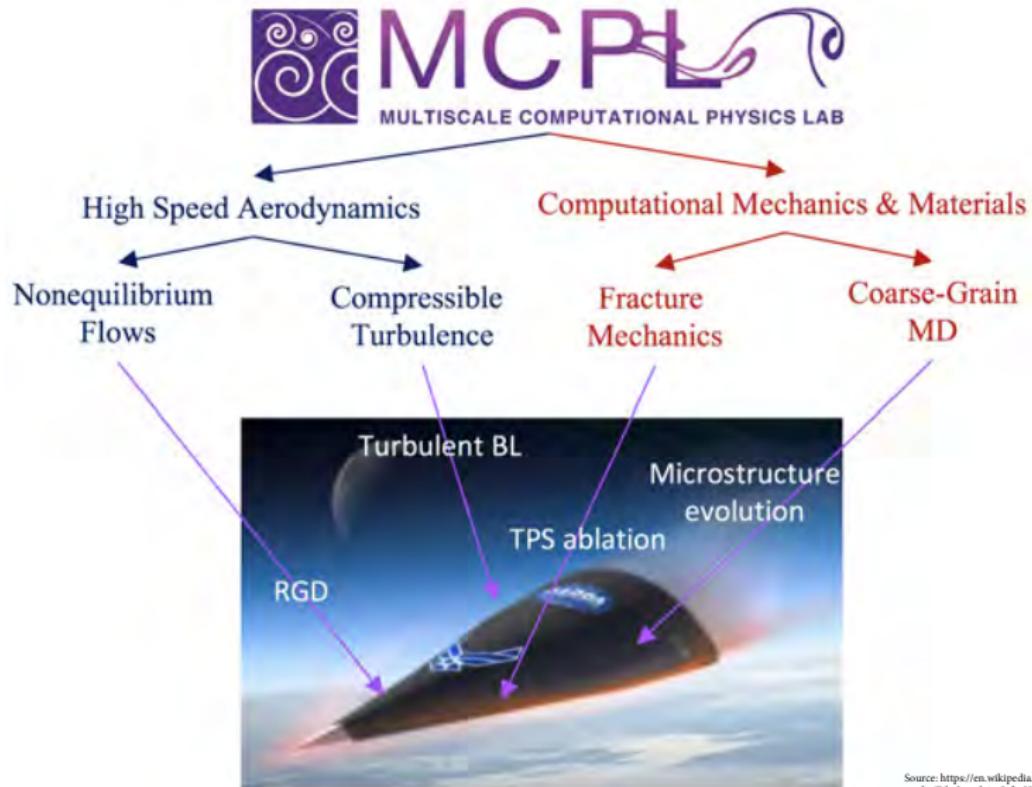
Source: James Chen, University at Buffalo

417.03 MPa

416.84 MPa (Physics-based Simulation)

(Walker, Sun, and Chen, accepted for publication)

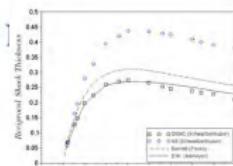
# Design of Hypersonic Vehicles



# Design of Hypersonic Vehicles

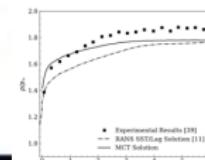


## High Speed Aerodynamics

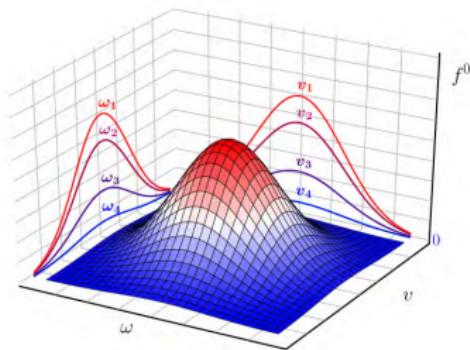
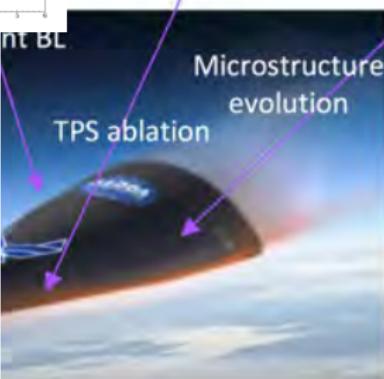


## Computational Mechanics & Materials

### Fracture Mechanics



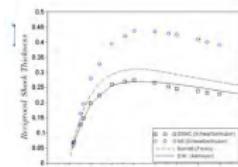
### Coarse-Grain MD



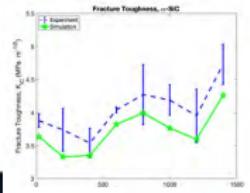
# Design of Hypersonic Vehicles



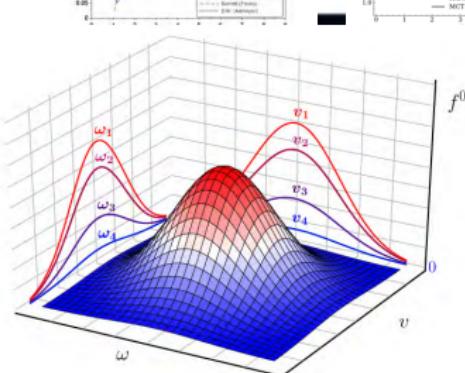
## High Speed Aerodynamics



## Computational Mechanics & Materials



## Coarse-Grain MD



Heat  
Conduction

Elasticity

$T$

$\dot{u}_i$

$\nu(T, \xi)$

$\sigma_{ij}^E$

$\psi^+$

$\psi^-$

$\beta^D(\xi, \eta) \Rightarrow \varepsilon_{ij}^D$

$k(T, \xi)$

$\xi$

$f_{therm}(\xi, T) = 1/2k(T, \xi)T_jT_i$

$\eta$

$f_{twinning}(\xi, \eta)$

Twinning

# Acknowledgments

- Dr. Paul DesJardin
- Dr. David Salac
- Dr. Matt McGurn
- Dr. Eric Walker
- Dr. Rozie Zangeneh
- Mae Sementilli
- Jason Sun
- Joe Marziale
- Yu Chen



The relentless pursuit of hypersonic flight

How much new science will it take to design a vehicle that can routinely fly at many times the speed of sound? (I. Leyva)



MCPL] THANKS [YOU!!!



James Chen; (716) 645-3162; chenjm@buffalo.edu

Source: James Chen, University at Buffalo